Math 1281 Name:
September 25, 2003

Midterm Exam I

This is a closed book, closed notes exam. Calculators are allowed. Work all problems. The first 2 problems are multiple choice. Please circle the correct answer. (There will be no partial credit for these 2 problems). Problems 3-7 are free response. For these problems please do your work in the space provided and show all work. Partial credit will be given. However a correct answer may not receive full credit if the justification is incomplete or incorrect. If you need extra space, work on the back of the pages. Please clearly label all work. There are 100 points and 7 problems on this exam

1. (10) When plotted on a log-linear scale, the function $y=f(x)$ is a straight line whose slope is 2 and whose y-intercept is –2. More precisely, x and y are related by the equation

$$\log_{10}(y) = 2\ x - 2.$$

Which of the following expresses the functional relation between x and y?

(a) $y =- 2(10^{2x})$

(b) $y=\frac{1}{100} . 10^{2x}$ XXXXXX

(c) $y=-2\ x^2$

(d) $y=\frac{1}{100} . x^2$

(e) None of the above.

2. (10) The graph of the function *$y=f(x)=2x^3 - 3x^2$* is a cubic with a local maximum at *(x,y)=(0,0)* and local minimum at *(x,y) =(1,-1)* as shown in the graph to the right. Where are the local maximum and local minimum of the cubic *$y=f(x-1)+3$*?

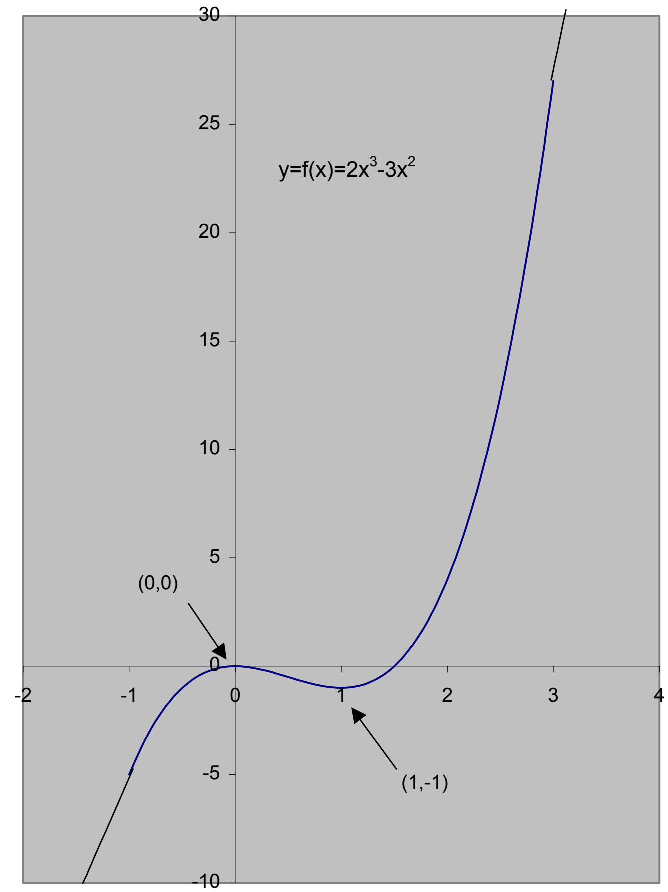
(a) (1,3), (2,2)   XXXXX

(b) (0,3) , (1,2)

(c) (-1,-3), (0,2)

(d) (-1,-3), (0,-4)

(e) None of the above

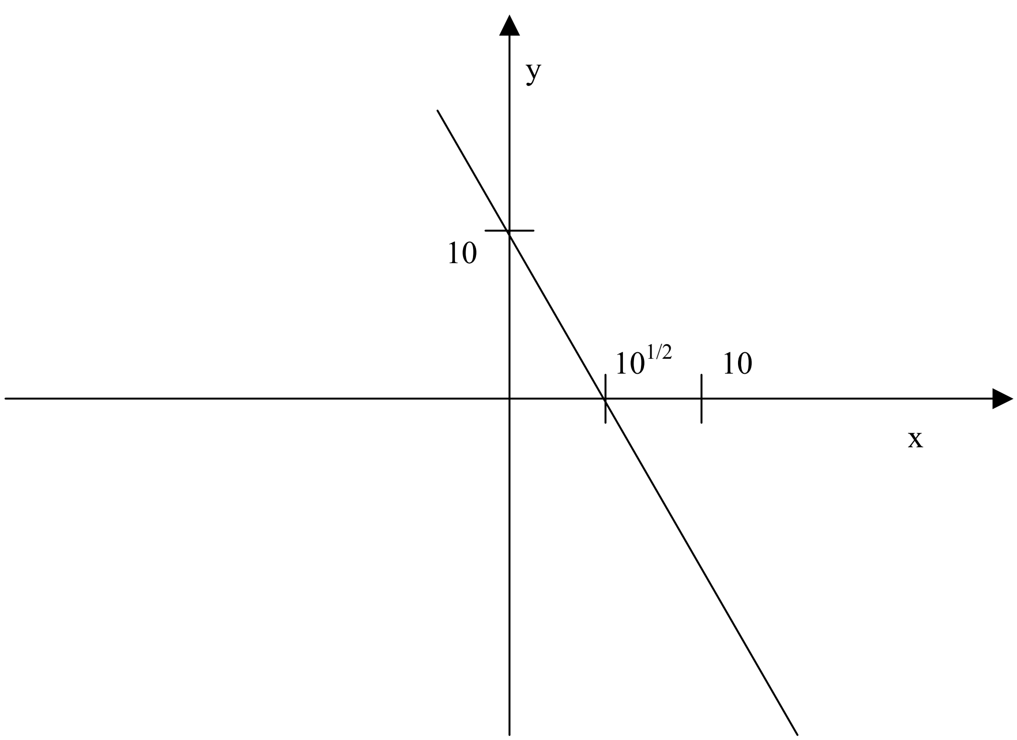


3. (15)

a. (7) Use a logarithmic transformation, base 10, to find a linear relationship between the quantities y = $10\,x^{-2}$.

$\log\ y = 1 - 2 \log x$

b. (8) Graph the resulting linear relationship on a log-log plot and indicate clearly the coordinates of the points at which the curve cuts the coordinate axes.



4. (10) Compute the following limits, if they exist.

a. (5) $$\lim_{n\to\infty}\frac{4n^2-n}{2n^2+1} = \lim_{n\to\infty}\frac{4-\frac{1}{n}}{2+\frac{1}{n^2}} = \frac{4-0}{2+0} = \frac{4}{2} = 2.$$

b. (5) $\lim_{n\to\infty}\frac{(-1)^n}{2n+1} = 0$, because the numerator stays bounded as $n \to \infty$ while the denominator grows without bound.

5. (15) A strain of bacteria reproduces asexually every 25 minutes. That is, every 25 minutes, each bacteria cell splits into two cells.

a. (5) Write down the recursion that describes this bacterial reproduction and find a solution for the recursion. Label the variables that you use.

Let $N(t)$ be the number of bacteria after t units of time, i.e., the number of bacteria after 25 t minutes have elapsed from some reference time $t=0$. The recursion is

$$N(t+1)=2\,N(t).$$

Assuming $N(0)$ bacteria at time $t=0$, the general solution is

$$N(t)=N(0)\,2^t, \text{ for integer } t.$$

b. (10) If there are now exactly 640 bacteria, how long ago were there 10 bacteria?

Suppose there were 10 bacteria at time $t=0$, so $N(0)=10$. Now, at time t, there are $N(t)=640$ bacteria. Thus $N(t)=N(0)\,2^t$ or $640 = 10\,(2^t)$. Thus $2^t = 64$ or $t = 6$ units. Thus there were 10 bacteria $25\,t = (25)(6)= 150$ minutes ago.

6. (20) Suppose $f(x) = \frac{x+5}{x-1}$ with natural domain.

a. (5) Determine the domain of *f*.

The domain of f is { x ε R : x ≠ 1 }

b. (5) Determine the range of *f*.

y is in the range of f if and only if y = (x+5)/(x-1) for some real x, x ≠ 1. Solving for x in this relation we have y(x-1)= x+5 or x(y –1)= y +5. Thus x=(y+5)/(y-1). Thus any y ≠ 1 can be expressed as y= $f(x) = \frac{x+5}{x-1}$. Note that y=1 is not in the range, since there is no x such that 1 = (x+5)/(x-1).

Thus the range of f is { y ε R : y ≠ 1 }

c. (10) Find f^{-1}, and give its domain and range.

From part b., f^{-1}(y) = x=(y+5)/(y-1).
The domain of f^{-1} = the range of f: { y ε R : y ≠ 1 }
The range of f^{-1} = the domain of f: { x ε R : x ≠ 1 }

7. (20) Solve for the variable:

a. (6) $\ln\left(x^2+1\right)=3$

$x^2 + 1 = e^3$, so there are two solutions: $x = \sqrt{e^3 - 1}$ and $x = -\sqrt{e^3 - 1}$

b. (6) $e^x = 10^{2x-1}$

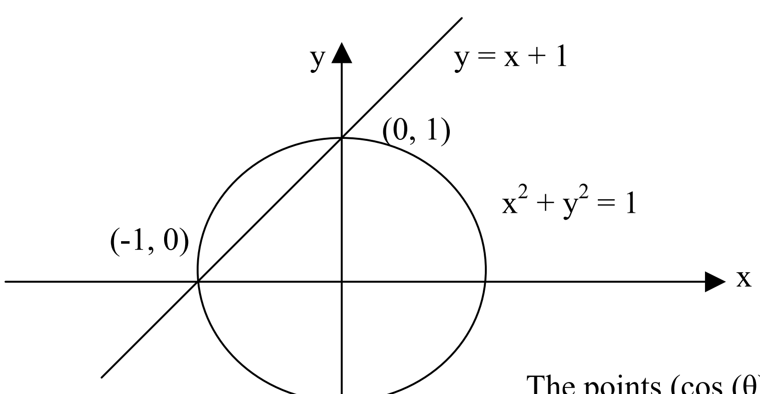
Method 1: Use natural logs.
$\ln(e^x) = \ln(10^{2x-1})$, so $x = (2x-1)\ln(10)$. Thus $(2\ln(10) - 1)\,x = \ln(10)$, or

$x = \ln(10)/(2\ln(10) - 1)$

Method 2: Use log base 10.
$\log(e^x) = \log(10^{2x-1})$, so $x\log(e) = 2x-1$. Thus $(2 - \log(e))\,x = 1$, or

$x = 1/(2 - \log(e))$

c. (8) $\sin(\theta) = \cos(\theta) + 1$, for $0 \le \theta < 2\pi$. (Hint: Use the definition of these functions via the points $(\cos(\theta), \sin(\theta))$ on the unit circle. You can characterize the solutions of this equation as the intersection points of a straight line with the unit circle. A straight line can intersect a circle in at most two points.)



The points $(\cos(\theta), \sin(\theta))$ lie on the intersection of the line $y = x + 1$ and the unit circle. The intersection points are $(\cos(\theta), \sin(\theta)) = (-1, 0)$, so $\theta = \pi$, and $(\cos(\theta), \sin(\theta)) = (0, 1)$, so $\theta = \pi/2$.