Math 1281 Name:
October 30, 2003 Section:

Midterm Exam II

This is a closed book, closed notes exam. Calculators are allowed. Work all problems. The first 2 problems are multiple choice. Please circle the correct answer. (There will be no partial credit for these 2 problems). Problems 3-7 are free response. For these problems please do your work in the space provided and show all work. Partial credit will be given. However a correct answer may not receive full credit if the justification is incomplete or incorrect. If you need extra space, work on the back of the pages. Please clearly label all work. There are 100 points and 7 problems on this exam The number of points for each problem is listed immediately after the problem number.

(10) If $f(x) = x - \sin x$, what is $f''(x)$?

(a) $x - \cos x$

(b) $1 + \sin x$

(c) $-\sin x$

(d) $\sin x$ X

(e) None of the above.

2. (10) Suppose that the function $f(x)$ is differentiable and that $f(1) = 2,\ f(2) = 3,$ $f'(2) = 3,\ f'(1) = 5.$ Let $H(x) = \left(3x^2 - 1\right)f(x) + 2$. Which of the following statements about $H'(x) = \dfrac{dH}{dx}$ is true?

(a) $H'(1) = 4$

(b) $H'(2) = 50$

(c) $H'(2) = 33$

(d) $H'(1) = 22$ X

(e) None of the above.

$H'(x) = 6xf(x) + (3x^2 - 1)f'(x)$
$H'(1) = 6f(1) + 2f'(1) = (6)(2) + (2)(5) = 22$
$H'(2) = 12f(2) + 11f'(2) = 36 + 33 = 69$

3. (15) Compute the following limits, if they exist. Show your work.

a. (5) $$\lim_{x\to\infty}\frac{e^{-2x}-1}{e^{-x}-1}=\lim_{x\to\infty}\frac{\frac{1}{e^{2x}}-1}{\frac{1}{e^{x}}-1}=\frac{\lim_{x\to\infty}(\frac{1}{e^{2x}}-1)}{\lim_{x\to\infty}(\frac{1}{e^{x}}-1)}=\frac{0-1}{0-1}=1$$

b. (5) $$\lim_{x\to 3}\frac{x^2-5x+6}{x-3}=\lim_{x\to 3}\frac{(x-3)(x-2)}{x-3}=\lim_{x\to 3}(x-2)=3-2=1$$

c. (5) $\lim_{h\to 0}\frac{e^h-1}{h^{2/3}}$, given that $\lim_{h\to 0}\frac{e^h-1}{h}=1$,

$$\lim_{h\to 0}\frac{e^h-1}{h^{2/3}}=\lim_{h\to 0}h^{1/3}(\frac{e^h-1}{h})=\lim_{h\to 0}h^{1/3}\lim_{h\to 0}(\frac{e^h-1}{h})=(0)(1)=0$$

4. (10) Suppose $W(t)$ denotes the amount of a radioactive material left after time *t*. Assume that $W(0) = 5$ and that $\frac{dW}{dt} = -2W(t)$.

a. (5) Verify in detail that $W(t) = 5e^{-2t}$ satisfies the differential equation and initial condition above.

$W(0) = 5e^{-(2)(0)} = (5)(1) = 5$

$\frac{dW}{dt} = \frac{d}{dt}(5e^{-2t}) = 5(-2)e^{-2t} = -10e^{-2t} = -2(5e^{-2t}) = -2W(t)$

b. (5) What is the half-life of this material?

Let *T* be the half-life. Then $W(T) = \frac{1}{2}W(0)$ so $5e^{-2T} = \frac{1}{2}(5)$. Thus $e^{-2T} = 2^{-1}$ and $T = \frac{\ln 2}{2}$.

5. (20) Consider the curve $y + y^3 = 3e^x + 7$. This is an implicit relation for y as a function of x.

a. (10) Compute $\frac{dy}{dx}$ when $(x, y) = (0,2)$.

$y' + 3y^2 y' = 3e^x$ so $(1 + 3y^2)y' = 3e^x$ and $y' = \frac{3e^x}{1 + 3y^2}$.

Thus $y'_{(0,2)} = \frac{3e^0}{1 + 3(2)^2} = \frac{3}{13}$

b. (5) Determine the equation of the tangent line to this curve at the point $(x, y) = (0,2)$.

$y - c = m(x - d)$ where *(c,d)=(0,2), m=3/13*

Thus $y - 2 = \frac{3}{13}x$

c. (5) Determine the equation of the normal line to this curve at the point (x, y)=(0,2) Recall that the slope of the normal is the negative reciprocal of the slope of the tangent.

$y - 2 = -\frac{13}{3}x$

6. (10) Use the method of linear approximation with the function $f(x) = \sqrt[3]{x}$ to estimate $\sqrt[3]{27.2}$. Recall that the linear approximation to $f(x)$ near the point $(a, f(a))$ takes the form $f(x) \approx f(a) + f'(a)(x-a)$.

a=27, $f(a) = \sqrt[3]{27} = 3$

$f'(x) = \frac{1}{3}x^{-2/3}$ so $f'(a) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$

$$f(27.2) \approx f(27) + f'(27)(27.2 - 27) = 3 + \frac{1}{27}(\frac{2}{10}) = 3 + \frac{2}{270}$$

7. (25) Compute the following derivatives. Simplify your answer, if appropriate

a. (6) $\frac{d}{dx}\left(xe^{-x}\right) = x(e^{-x})' + e^{-x}(x)' = -xe^{-x} + e^{-x} = e^{-x}(1-x)$

b. (7) $$\frac{d}{dx}\left(\frac{e^x+1}{2x+1}\right)=\frac{(2x+1)(e^x)'-(e^x+1)(2x)'}{(2x+1)^2}=\frac{e^x(2x-1)-2}{(2x+1)^2}$$

c. (6) $$\frac{d}{d\theta}(\ln(\cos\theta))=\frac{1}{\cos\theta}\frac{d}{d\theta}\cos\theta=-\frac{\sin\theta}{\cos\theta}=-\tan\theta$$

d. (6) $$\frac{d}{dx}\left(x^x\right)=\frac{d}{dx}e^{\ln x^x}=\frac{d}{dx}e^{x\ln x}=e^{x\ln x}\frac{d}{dx}(x\ln x)=e^{x\ln x}(\ln x+1)=x^x(1+\ln x)$$