Math 1281
December 4, 2003

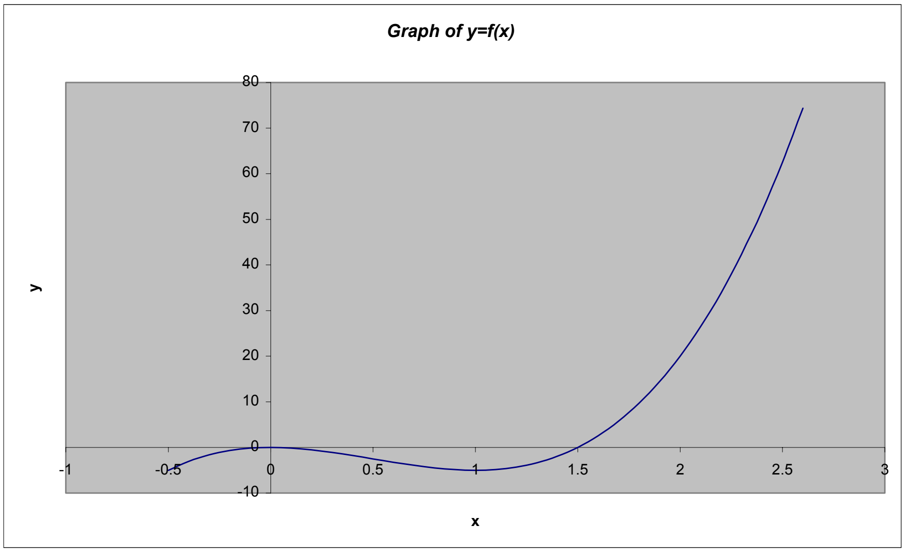
Name:
Section:

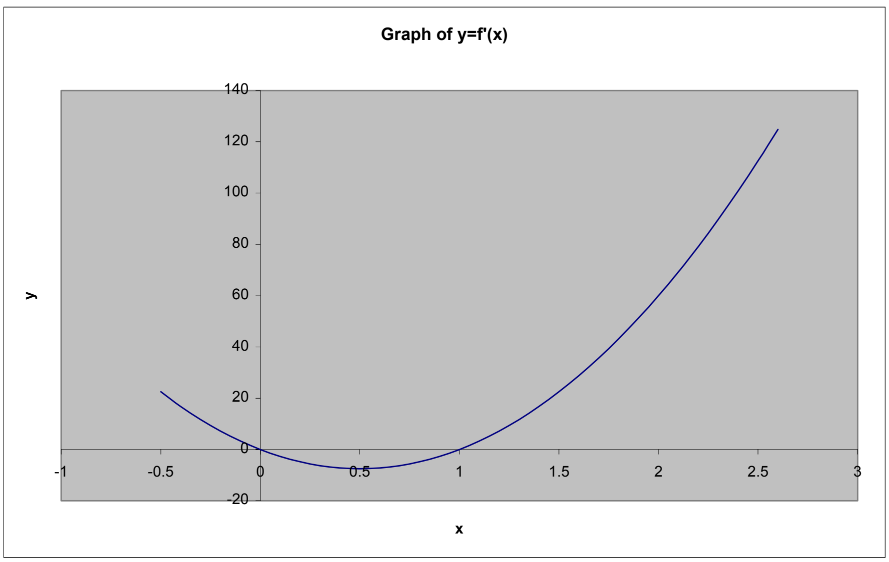
Midterm Exam III (Solutions)

This is a closed book, closed notes exam. Calculators are allowed. Work all problems. The first 4 problems are multiple choice. Please circle the correct answer. (There will be no partial credit for these 4 problems). Problems 5-9 are free response. For these problems please do your work in the space provided and show all work. Partial credit will be given. However a correct answer may not receive full credit if the justification is incomplete or incorrect. If you need extra space, work on the back of the pages. Please clearly label all work. There are 100 points and 9 problems on this exam. The number of points for each problem is listed immediately after the problem number.

Problem	Score	Possible
1		5
2		5
3		5
4		5
5		15
6		15
7		15
8		15
9		20
Total		100

Problems 1 and 2 refer to the figures to follow. On top is the graph of the function *f(x),* and on the bottom is the graph of the derivative $f'(x)$.



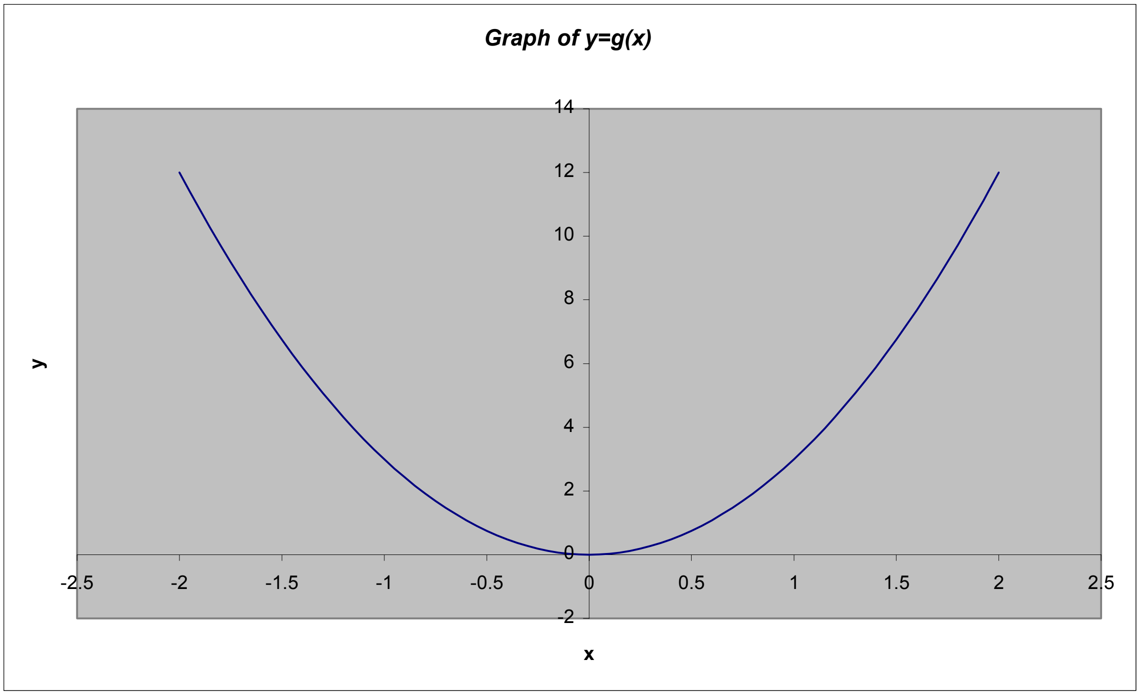


(5) 1. At $x=0$, the graph of $y=f(x)$ has a

a. zero and local minimum
b. zero and local maximum X
c. zero and absolute maximum
d. zero and inflection point
e. none of the above

(5) 2. At $x=0.5$, the graph of $y=f(x)$ has a

a. zero
b. local minimum
c. local maximum
d. inflection point X
e. none of the above



(5) 3. Above is a graph of the function *y=g(x)*. This function has two continuous derivatives. Which of the following statements is correct for all *x* shown in the graph?

a. $g'(x) > 0$
b. $g'(x) < 0$
c. $g''(x) \leq 0$
d. $g''(x) \geq 0$ X
e. $g'(x) \neq 0$

(5) 4. If $\int_{-1}^{3} f(x)dx = -2$ and $\int_{-1}^{1} f(x)dx = 3$, what is $\int_{3}^{1} f(x)dx$?

a. -5
b. -3
c. 2
d. -2
e. None of the above X (The integral equals +5.)

(15) 5. Find all asymptotes of

$$y = \frac{x^2 + 3}{x^2 - 1}.$$

For each asymptote, state whether it is horizontal, vertical or oblique. Justify your answer by showing the limits for the horizontal or oblique asymptotes. A graph alone is not sufficient.

Solution: Vertical asymptotes at $x = \pm 1$ because the denominator of the expression for y vanishes at these values. Since $y = 1 + \dfrac{4}{x^2 - 1}$ we have $\lim_{x\to\infty} y = 1$ and $y = 1$ is a horizontal asymptote. There are no oblique asymptotes.

(15) 6. Find the dimensions of a right-circular cylindrical can, open on the top and closed on the bottom, so that the can holds 1 cubic foot and uses the least amount of material. Verify that your answer yields the global minimum. Recall that the volume of a right-circular cylinder with radius r and height h is $V = \pi r^2 h$ and the surface area of the lateral sides of the cylinder is $S = 2\pi rh$.

Solution: $V = \pi r^2 h = 1$ so $h = \dfrac{1}{\pi r^2}$. Let A be the total surface area in square feet. Then $A(r) = \pi r^2 + 2\pi rh = \pi r^2 + \dfrac{2}{r}$ with domain $0 < r$. The problem is to minimize A on this domain. Since A blows up as r approaches 0 and r approaches ∞. The minimum must exist and occur at a critical point. We find $A'(r) = 2\pi r - \dfrac{2}{r^2}$, so $A'(r) = 0$ exactly when $r = \sqrt[3]{\dfrac{1}{\pi}}$. Further, $A''(r) = 2\pi + \dfrac{4}{r^3} > 0$ for all r in the domain of A. Thus the absolute minimum of A occurs at $r = \sqrt[3]{\dfrac{1}{\pi}}$. The optimal dimensions are the previously listed value of r and $h = r$.

(15) 7. Use l'Hospital's rule to find the limit.

$$\lim_{x \to 0} \frac{\sin 3x}{\ln(1-2x)}.$$

Show your work.

Solution: This is a $\frac{0}{0}$ limit, so L'H applies.

$$\lim_{x \to 0} \frac{\sin 3x}{\ln(1-2x)} = \lim_{x \to 0} \frac{3\cos 3x}{\left(\frac{-2}{1-2x}\right)} = \frac{3}{-2} = -3/2.$$

(15) 8. Set up but do not evaluate the definite integral for finding the area of the finite region bounded by the curves $y = 0$ and $y = 9 - x^2$. Be sure to include the limits of integration.

Solution: $9 - x^2 = 0$ if and only if $x = \pm 3$ so the curves intersect at (-3,0) and (3,0). Also $9 - x^2 \geq 0$ only for $-3 \leq x \leq 3$ and this function goes to $-\infty$ as $|x| \to \infty$. Thus the area A is given by $A = \int_{-3}^{3} (9 - x^2)\, dx$.

(20) 9. Evaluate the following definite and indefinite integrals. Show your work.

(10) a. $\int_0^{\pi/4} \sin(2x)dx$. Evaluate the integral *exactly*.

Solution: $\int_0^{\pi/4} \sin(2x)dx = -\frac{\cos(2x)}{2}\Big]_0^{\pi/4} = -(\frac{\cos(\pi/2)}{2} - \frac{1}{2}) = \frac{1}{2}$

(10) b. $\int (x^2 - 2)^2 dx$

Solution: $\int (x^2 - 2)^2 dx = \int (x^4 - 4x^2 + 4)\, dx = \frac{1}{5}x^5 - \frac{4}{3}x^3 + 4x + C$