

(5) 1. What is $\lim_{x \rightarrow \infty} \frac{\sqrt{\pi}}{1 + \pi e^{-2x}}$?

- (a) 0
- (b) $\frac{\sqrt{\pi}}{1 + \pi}$
- (c) $\frac{1}{\sqrt{\pi}}$
- (d) ∞
- (e) None of the above.

(5) 2. What is $\lim_{x \rightarrow 0} \frac{\sin(2x)}{4x}$?

- (a) 0
- (b) $\frac{1}{4}$
- (c) $\frac{1}{2}$
- (d) ∞
- (e) None of the above.

(5) 3. For which value of a is

$$f(x) = \begin{cases} 2x^2 & \text{for } x \leq 1 \\ 3x + a & \text{for } x > 1 \end{cases}$$

continuous for all real x ?

- (a) -1
- (b) 0
- (c) 1
- (d) $\frac{3}{2}$
- (e) None of the above.

(5) 4. Suppose that the function $f(x)$ is differentiable and that $f(2) = 1$, $f'(2) = \frac{1}{3}$, $f'(1) = 5$. Let $g(x) = f^{-1}(x)$. What is $g'(2)$?

- (a) $\frac{1}{5}$
- (b) $\frac{1}{3}$
- (c) 3
- (d) 5
- (e) Cannot be determined from the information given.

- (5) 5. Suppose that the function $f(x)$ is differentiable and that $f(2) = 1$, $f'(2) = \frac{1}{3}$, $f'(1) = 5$. Let $g(x) = f^{-1}(x)$. What is $g'(1)$?
- (a) $\frac{1}{5}$
 - (b) $\frac{1}{3}$
 - (c) 3
 - (d) 5
 - (e) Cannot be determined from the information given.

- (5) 6. On which of the following plots would a graph of the function $f(x) = 3x^{6/5}$ (vertical axis) as a function of x (horizontal axis) appear as a straight line?
- (a) linear scale on both the horizontal and vertical axes.
 - (b) logarithmic scale on both the horizontal and vertical axes.
 - (c) linear scale on the horizontal and logarithmic scale on the vertical axes.
 - (d) logarithmic scale on the horizontal and linear scale on the vertical axes.
 - (e) None of the above.

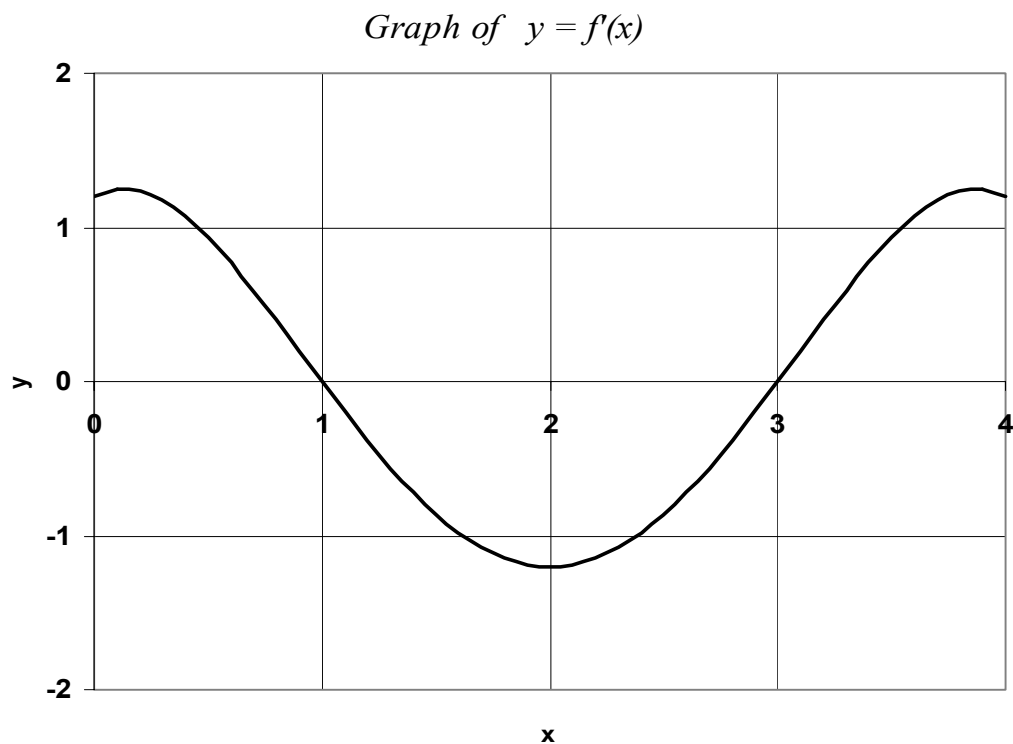
- (5) 7. Denote the leaf size y of a plant by $y = f(x)$, where x is the age of the leaf. Assume that

$$f(x) = 4x^{5/2} \quad \text{for } x \geq 0.$$

If you measure x with an accuracy of 5%, and if you compute y using the formula for $f(x)$, what is the percentage error of your computed y ?

- (a) 2 %
 - (b) 4 %
 - (c) 5 %
 - (d) $12\frac{1}{2}$ %
 - (e) None of the above.
- (5) 8. If $\int_1^3 f(x)dx = 3$ and $\int_2^3 f(x)dx = 1$, what is $\int_1^2 f(x)dx$?
- (a) -2
 - (b) -1
 - (c) 1
 - (d) 2
 - (e) None of the above.

Problems 9 and 10 refer to the graph below, which is the graph of the *derivative* $f'(x)$ of a function $f(x)$.



(Yes, the graph shows $f'(x)$ (the derivative of $f(x)$), but the questions are about the function $f(x)$).

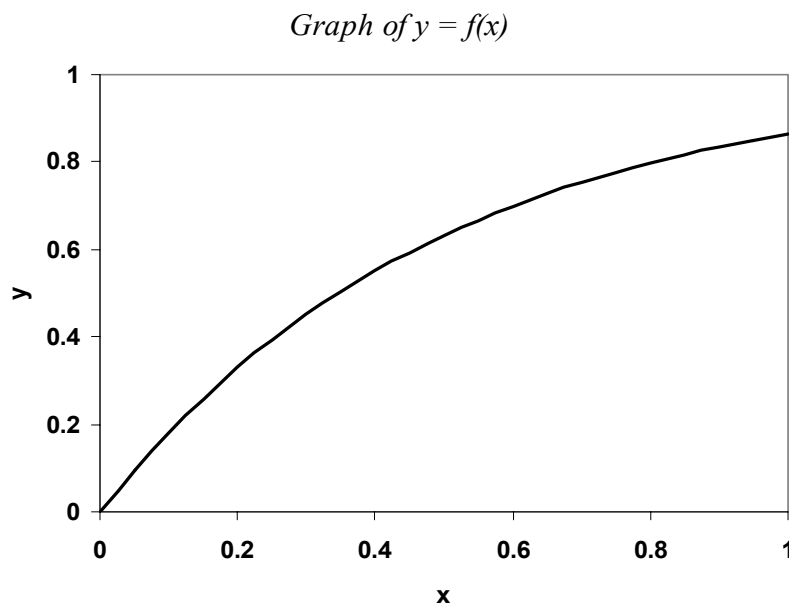
(5) 9. At $x = 2$, the graph of $y = f(x)$ has a

- (a) zero.
- (b) local minimum.
- (c) local maximum.
- (d) inflection point.
- (e) None of the above.

(5) 10. At $x = 3$, the graph of $y = f(x)$ has a

- (a) zero.
- (b) local minimum.
- (c) local maximum.
- (d) inflection point.
- (e) None of the above.

- (5) 11. Below is a graph of the function $y = f(x)$.



Which of the following statements is correct?

- (a) $f'(x) > 0$ and $f''(x) > 0$.
 - (b) $f'(x) > 0$ and $f''(x) < 0$.
 - (c) $f'(x) < 0$ and $f''(x) > 0$.
 - (d) $f'(x) < 0$ and $f''(x) < 0$.
 - (e) None of the above.
- (5) 12. If the size of a population at time t is denoted by $N(t)$ and if the growth rate of the population is given by

$$g(N) = N(N-1)(3-N),$$

which of the following differential equations describes the per capita growth rate of this population?

- (a) $\frac{1}{N} \frac{dN}{dt} = (N-1)(3-N)$
- (b) $\frac{1}{N} \frac{dN}{dt} = g'(N)$
- (c) $\frac{dN}{dt} = (N-1)(3-N)$
- (d) $\frac{dN}{dt} = \log(g(N))$
- (e) None of the above.

(20) 13. Compute the following limits. Give an exact answer and show your work.

(10) a. $\lim_{x \rightarrow 0} \frac{\sin x}{x + x^2}$

(10) b. $\lim_{x \rightarrow \infty} x e^{-3x}$

(20) 14. For each function, compute the derivative. Show your work.

(10) a. $f(x) = x^5 + \frac{1}{x^2} - \sqrt[3]{x}$

(10) b. $f(x) = x^{-3} e^{2x}$

(30) 15. For each function, compute the derivative. Show your work.

(15) a. $f(x) = \frac{3x^2}{7 + 5x^2}$

(15) b. $f(x) = x^5 + 5^x$

(30) 16. For each function, compute the derivative. Show your work.

(15) a. $f(x) = \exp(\tan(x^2 + 1))$

(15) b. $f(x) = \ln(\cos^2 x)$

(10) 17. Find the derivative of

$$f(x) = g(-\ln x),$$

where $g(x)$ is a differentiable function.

- (15) 18. Find $\frac{dy}{dx}$ when

$$y = \ln(x + y^2)$$

Write your answer as $\frac{dy}{dx} = \dots$, and simplify.

- (15) 19. Consider the function

$$f(x) = (1 + 2x)^k$$

where k is a positive integer.

- (10) a. Find the linear approximation of $f(x)$ near $x = 0$.

- (5) b. Use your answer in part (a) to estimate $(1.002)^{50}$. Show your work and make sure that we can recognize that you used part (a).

- (15) 20. Find all the asymptotes of

$$y = \frac{2x+1}{x+1}.$$

For each asymptote, state whether it is horizontal, vertical, or oblique, and justify your answer.

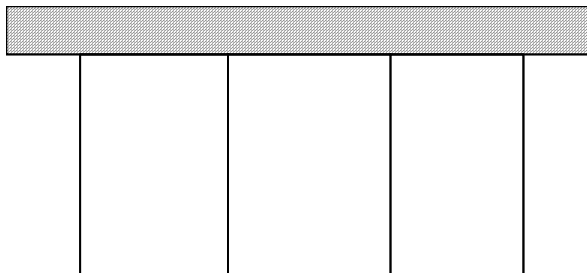
- (20) 21. Let

$$f(x) = 7 + 5x - x^3$$

and note that $f'(x) = 5 - 3x^2$, and that $f''(x) = -6x$.

- (10) a. Determine where $f(x)$ is increasing and where it is decreasing. Find all local extrema and identify whether they are local maxima or local minima.
- (10) b. Determine where $f(x)$ is concave up and where it is concave down. Find all inflection points.

- (15) 22. Sven and Ole want to fence in a rectangular plot of 1600 square feet against a solid wall. They also want to use additional fencing to build two internal dividers parallel to two outer boundary sections, as shown in the figure below. What is the minimum total length of fencing that this project will require? Verify that your answer yields the global minimum.



- (10) 23. The exponential growth function is given by

$$N(t) = N_0 e^{rt} \quad \text{for } t \geq 0,$$

where $N(t)$ denotes the amount of material left at time t and r is the growth rate ($r > 0$). Show that $N(t)$ satisfies the differential equation

$$\frac{dN}{dt} = rN.$$

- (20) 24. Solve the initial value problem

$$\frac{dy}{dx} = e^{-0.5x} \quad \text{with} \quad y(0) = 5.$$

- (10) 25. Suppose that $N(t)$ is the population at time t . Assume that

$$\frac{dN}{dt} = 4N \left(1 - \frac{N}{10} \right) \quad \text{for } t \geq 0,$$

and that $N(0) = 5$. Use a linear approximation to estimate $N(0.1)$.

- (10) 26. Evaluate $\int \sin\left(\frac{\pi}{4}x\right)dx$.

- (20) 27. Use a formula from geometry to find the **exact value** of the definite integral

$$\int_{-3}^3 (\sqrt{9-x^2} - 3) dx .$$

- (15) 28. Find the **exact value** of the definite integral $\int_0^4 e^{-4x} dx$. Show your work.

- (15) 29. Find the function given by $f(x) = \frac{d}{dx} \int_1^{x^2} (1+u^2) du$. Simplify your answer.

- (15) 30. Find the **exact value** of the definite integral $\int_0^{\pi/8} \sec^2(2x) dx$. Show your work.

- (20) 31. Evaluate the following indefinite integrals.

(10) a. $\int \frac{5}{x+4} dx$.

(10) b. $\int 4xe^{x^2} dx$.

- (15) 32. Set up but do not evaluate the integral for finding the area of the region bounded by curve $xy = 1$ and the line $x + 2y = 3$.