## Math 1281

## Midterm Exam <br> October 31, 2002

Closed book, closed notes. Simple calculators are allowed. Work all problems. The first 2 problems are multiple choice. Please circle the correct answer. Only the correct answer will receive credit -- there is no partial credit. Problems 3-7 are free response. Please work the problem in the space provided, and show all work. Partial credit will be given. A correct answer may receive less than full credit if the work is incomplete or incorrect. If you need extra space, work on the back of the pages, or attach extra sheets. Please clearly label all work.

| Problem | Score | Possible |
| :---: | :---: | :---: |
| 1 |  | 10 |
| 2 |  | 10 |
| 3 |  | 15 |
| 4 |  | 20 |
| 5 |  | 15 |
| 6 |  | 15 |
| 7 |  | 15 |
| Total |  | 100 |

(10) 1. What is $\frac{d^{2}}{d x^{2}}(\cos 2 x)$ ?
(a) $-2 \sin 2 x$
(b) $-4 \sin 2 x$
(c) $-4 \cos 2 x$
(d) $4 \cos 2 x$
(e) None of the above.
(10) 2. Suppose that the function $f(x)$ is differentiable and that $f(1)=2, f(2)=3, f^{\prime}(2)=4$, $f^{\prime}(1)=5$. Let $g(x)=f^{-1}(x)$. Which of the following statements is true?
(a) $\quad g^{\prime}(1)=\frac{1}{2}$
(b) $g^{\prime}(2)=\frac{1}{3}$
(c) $\quad g^{\prime}(3)=\frac{1}{4}$
(d) $\quad g^{\prime}(4)=\frac{1}{5}$
(e) None of the above.
(15) 3. Compute the following limits.
(5) a. $\lim _{x \rightarrow 0} \frac{2^{x}-1}{2 x}$
(5) b. $\lim _{x \rightarrow \infty} \frac{e^{x}}{1-e^{x}}$
(5) c. $\lim _{x \rightarrow 0} \frac{\sin 2 x}{x}$
(20) 4. A population of bacteria is growing exponentially. Let $M(t)$ be the mass of the population measured in grams, where $t$ is the elapsed time since the beginning of the experiment, measured in hours. At the beginning of the experiment, the mass of the population, $M(0)$, is $10^{-3}$ grams. Three hours later, the mass of the population, $M(3)$, is $1.5 \times 10^{-3}$ grams. Assume that the same exponential growth continues indefinitely.
(10) a. Compute $\frac{d M}{d t}$ exactly five days after the beginning of the experiment.
(10) b. Compute $\frac{d M}{d t}$ when the mass of the population reaches $6 \times 10^{21}$ metric tons (1 metric ton $=10^{6}$ grams).
(15) 5. Consider the curve $y+\ln y+x=2 x^{2}$. Compute $\frac{d y}{d x}$ and $\frac{d^{2} y}{d x^{2}}$ when $(x, y)=(1,1)$.
(15) 6. Use the method of linear approximation to estimate $\sin \left(31^{\circ}\right)$. Hint: $\sin \left(30^{\circ}\right)=\frac{1}{2}$.
(15) 7. Compute the following derivatives. Simplify your answer.
(5) a. $\frac{d}{d \theta}\left(\frac{1}{1+\cos \theta}\right)$
(5) b. $\frac{d}{d x}\left(\frac{e^{x}-e^{-x}}{x-1}\right)$
(5) c. $\frac{d}{d \theta}((\sin \theta)(\ln (\sin \theta)))$

