

Symbol Index

This index lists symbols used frequently in the text, followed by the sections in which they first appear.

$ \alpha $	Absolute value, 5.1.
A^*	Adjoint of the operator A , 3.7.
$\text{Ad } A, \text{Ad } \alpha$	Adjoint rep operators, 5.1, 5.6.
B_α	Algebra of permutation operators, 4.3.
a, a^*	Annihilation and creation operators, 9.7.
$A(G)$	Automorphism group of G , 1.3.
$\mathfrak{g}^+, \mathfrak{g}^-, \mathfrak{g}^3$ or J^+, J^-, J^3	Basis for a rep of $sl(2)$, 5.10.
$\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3$	Basis for $so(3)$, 7.1.
$J_n(z) = [(z/2)^n/n!] \sum_{k=0}^{\infty} [(-z^2/4)^k/(n+1)_k k!]$, $n = 0, 1, \dots$	Bessel function, 10.2.
A_α	Bisymmetric transformations, set of, 4.3.
$\varepsilon_{kl}, \varepsilon_\alpha$	Branch, 9.1.
Γ_n, Γ_n^m	Bravais lattices, 2.8.
h_m	Cartan subalgebra of a classical group, 9.1.
$\chi(g), g \in G$	Character of the group G , 3.4.
$C_\infty, U(1), SO(2)$	Circle group, 2.3.
CG	Clebsch–Gordan, 3.5.
$(\mu_i, \nu_j \xi sl)$	Clebsch–Gordan (CG) coefficients, 3.5.
$C(u, m; v, n w, k)$	Clebsch–Gordan coefficients for $SU(2)$, 7.7.
α_j	Clifford algebra, elements of, 9.6.
$C(T)$	Column permutations of a Young tableau, 4.2.
$[A, B], [\alpha, \beta]$	Commutator bracket, 5.1, 5.3.
\mathbb{C}	Complex numbers, 1.1.
$\bar{A} = (\bar{A}_{ij})$	Complex conjugate matrix, 3.1.
\mathcal{C}_m^n	Conjugacy class in a point group, 3.6.

- $\mathbf{u} \times \mathbf{v}$ Cross product, 3.8.
 C_n Cyclic group of order n , 2.4.
 $C_{\infty v}, C_{\infty h}$ Symmetry groups, 2.9, 7.6.
 $\Delta(\epsilon_1, \dots, \epsilon_m) = \prod_{j>k} (\epsilon_j - \epsilon_k)$, 9.2.
 $f'(x), \dot{f}(t)$ Derivative of a function, 5.1.
 $\dot{A}(t)$ Derivative of a matrix-valued function, 5.1.
 $\det A$ Determinant of matrix A , 2.1.
 $|\epsilon^{l_1}, \dots, \epsilon^{l_m}| = \sum_{s \in S_m} \delta_s \epsilon_{s(1)}^{l_1} \cdots \epsilon_{s(m)}^{l_m}$, Determinant, 4.4, 9.2.
 D_n Dihedral group, 2.4.
 $\dim V$ Dimension of vector space V , 3.2.
 $G \times H$ Direct product of groups, 1.5.
 $T_1 \oplus T_2$ Direct sum of reps T_1 and T_2 , 3.2, 3.5.
 $V \oplus W$ Direct sum of vector spaces V and W , 3.2, 3.5.
 $D_{\infty h}$ Symmetry group, 2.9.
 $T \cong T'$ Equivalence of reps, 3.1.
 $E(n), (E^+(n))$ Euclidean group in n -space (proper Euclidean group), 2.2.
 $\{a, O\}$ Euclidean group element, 2.2.
 (φ, θ, ψ) Euler angles, 7.1.
 $\exp \alpha: \mathfrak{g} \rightarrow G$ Exponential mapping, 5.5, 5.9.
 $\exp A = e^A = \sum_{j=0}^{\infty} A^j/j!$ Exponential of a square matrix, 5.1.
 G/N Factor group, 1.2.
 $\Gamma(z) = \lim_{n \rightarrow \infty} [n! n^z / (z)_{n+1}]$, $\Gamma(z+1) = z\Gamma(z)$, $\Gamma(n+1) = n!$, $n = 0, 1, 2, \dots$, Gamma function, 7.5.
 $GL(n, \mathbb{C}), GL(n, R), GL(n)$ General linear groups, 1.1, 5.4.
 G Group, 1.1.
 R_G Group algebra (ring) of the group G , 3.1, 3.3.
 $x = \sum_{g \in G} x(g) \cdot g$ Group algebra element, 3.1.
 $\varphi(g, h)$ Group product in local Lie group, 5.2.
 $T(g), T(A)$ Group rep matrices, 3.1.
 $\mathbf{T}(g), \mathbf{T}(A)$ Group rep operators, 3.1.
 H Hamiltonian operator, 3.8.
 $H_n(x) = (-1)^n \exp(x^2)(d^n/dx^n) \exp(-x^2)$, $n = 0, 1, 2, \dots$, Hermite polynomials, 10.1.
 \mathfrak{H} Hilbert space, 3.8, Appendix.
 $\mu: G \rightarrow G'$ Homomorphism of groups, 1.3.
 $\tau: \mathfrak{g} \rightarrow \mathfrak{g}'$ Homomorphism of Lie algebras, 5.3.
 ${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} [(a)_n (b)_n / (c)_n] (z^n/n!)$, $|z| < 1$, Hypergeometric series (see Pochhammer symbol), 7.2.
 Y Icosohedral group, 2.4.
 \mathfrak{g} Ideal, 4.3.
 $\mathcal{O}_\alpha, \mathcal{R}_\alpha$ Ideals in the group ring of S_α , 4.3.
 e Identity element in a group, 1.1.
 E_n Identity matrix, $n \times n$, 2.1, 3.3.
 E Identity operator, 2.1.
 T^σ Induced rep, 3.5.
 $\langle \mathbf{u}, \mathbf{v} \rangle, (\mathbf{u}, \mathbf{v})$ Inner product, 2.1, Appendix.
 $\int f(x) dx$ Integral of $f(x)$, 3.1, 3.8, Appendix.
 $dA, d_l A, d_r A, \delta A$ Invariant measures on a linear Lie group, 6.1.
 $I = -E$ Inversion operator, 2.3.
Irred Irreducible (representation), 3.2.
 $T(\mu)$ Irred rep of group G , indexed by the integer μ , 3.3.

- $[f_1, \dots, f_m]$ Irred reps of $GL(m)$, $U(m)$, and $SL(m)$, 9.1.
 $D^{(u,v)}$ Irred rep of proper homogeneous Lorentz group, 8.3.
 $D_+^{(u,v)}, D_-^{(u,v)}$ Irred reps of L^\dagger , 8.3.
 $D_+^{(l)}, D_-^{(l)}$ Irred reps of $O(3)$, 7.6.
 $D^{(u)}$ Irred reps of $SU(2)$, $SO(3, R)$, and $SL(2, \mathbb{C})$, 7.2, 7.3.
 $G \cong H$ Isomorphism of groups, 1.1, 2.4.
 G^x Isotropy subgroup of G at x , 1.4.
 $\delta_{ij} = 1$ if $i = j$, $= 0$ if $i \neq j$; Kronecker delta, 1.1.
 $L_n^{(\alpha)}(z) = [\Gamma(\alpha + n + 1)/\Gamma(\alpha + 1)n!] \sum_{j=0}^{\infty} [(-n)_j/(\alpha + 1)_j](z^j/j!)$, $n = 0, 1, 2, \dots$,
 Laguerre polynomial, generalized, 7.5, 10.2.
 Δ Laplacian, 3.1.
 $L_2(G)$ Lebesgue square-integrable functions on the group G , 6.2, Appendix.
 $L_2(\mathfrak{M})$ Lebesgue square-integrable functions on domain \mathfrak{M} , Appendix.
 $L(g)$ Left regular rep, 1.4, 3.1.
 $P_\mu^m(\cos \theta) = P_\mu^{0, -m}(\cos \theta)$, Legendre function (associated), 7.2.
 $P_n(\cos \theta) = {}_2F_1(n + 1, -n; 1; \frac{1}{2}(1 - \cos \theta))$, $n = 0, 1, 2, \dots$, Legendre polynomial, 7.2.
 \mathfrak{G} Lie algebra, 5.3.
 $L(G)$ Lie algebra of local Lie group G , 5.3.
 $so(3, 1)$ Lie algebra of homogeneous Lorentz group, 8.1, 9.10.
 L_α, D_α Lie derivatives, 5.9.
 $T: V \rightarrow V$ Linear transformation, 2.1, 3.1.
 $\ln A$ Logarithm of a matrix, 5.1.
 L^\dagger Lorentz group (complete), 8.3.
 $L(4)$ Lorentz group (homogeneous), 8.1.
 $L^{\dagger+}$ Lorentz group (proper), 8.1.
 $T = (T_{ij})$ Matrix, 2.1.
 ${}^{2S+1}L_J$ Multiplet, 9.8.
 $\nu(\mathbf{x}, g)$ Multiplier, 5.9.
 $\|\mathbf{u}\|, \|A\|$ Norm of a vector (operator), 2.1, 5.1, Appendix.
 N_A Null space of linear operator A , 3.3.
 O Octahedral group, 2.4.
 $n(G)$ Order of group G , 1.1.
 W^\perp Orthogonal complement of subspace W , 3.2.
 $O(n, \mathbb{C}), O(n, R), O(n)$ Orthogonal groups, 5.4.
 $\{\lambda_j\} \boxtimes \{\mu_k\}$ Outer product of irred reps of S_n and S_m , 9.9.
 δ_s Parity of the permutation s , 4.1, 9.2.
 $\partial f/\partial x, \partial_x f, f_x$ Partial derivative, 3.1.
 $\partial_{\pm 1/2, \pm 1/2}$ Partial derivatives, 8.4.
 $\{f_1, \dots, f_n\}$ Partition or frame, 4.1.
 $\begin{pmatrix} 1 & 2 & \cdots & n \\ p_1 p_2 & \cdots & p_n \end{pmatrix}, (p_i \cdots p_j), \sigma(x)$, Permutations, 1.1, 4.1.
 \perp Perpendicular, 2.8.
 $h = 2\pi\hbar \sim 1.054 \times 10^{-27}$ erg sec, Planck's constant, 3.1, 3.8.
 $(a)_n = a(a + 1) \cdots (a + n - 1) = \Gamma(a + n)/\Gamma(a)$, Pochhammer's symbol (see Bessel, hypergeometric, and Laguerre functions).
 P Poincaré group, 8.2.
 \mathcal{P} Poincaré group (covering group of), 8.4.
 $C_{nh}, C_{nv}, D_{nh}, D_{nd}$,
 $O_h = O \cup IO, S_{2n}, T_d$,
 $T_h = T \cup IT, Y_h = Y \cup IY$, Point groups, 2.5.

- P** Projection operator, 3.3, 3.7.
 R_A Range of linear operator A , 3.3.
 R_n Real Euclidean n -space or group ring of S_n , 2.1, 4.2.
 R Real numbers, 1.1.
 σ_k Reflection in plane perpendicular to k , 2.1.
Rep Representation, 3.1.
 $T|W$ Restriction of T to W , 3.2, 3.5.
 $R(g)$ Right regular rep, 3.1.
 $\alpha(\mathcal{H})$ Root form, 9.1.
 $C_k(\theta)$ Rotation through angle θ about axis k , 2.1.
 $S_k(\theta) = \sigma_k C_k(\theta)$, Rotation-inversion, 2.1.
 $R(T)$ Row permutations of a Young tableau, 4.2.
 $Spin(m)$ Simply connected covering group of $SO(m, \mathbb{C})$, 9.6.
 $SL(n, \mathbb{C}), SL(n, R), SL(n)$
 $[sl(n, \mathbb{C}), sl(n, R), sl(n)]$, Special linear groups (Lie algebras), 1.1, 5.4.
 $SO(n, \mathbb{C}), SO(n, R), SO(n)$
 $[so(n, \mathbb{C}), so(n, R), so(n)]$, Special orthogonal groups (Lie algebras), 1.4.
 $SU(m) [so(m)]$ Special unitary group (Lie algebra), 5.4.
 **$Y_l^m(\theta, \varphi) = [(2l + 1)(l - m)!/4\pi(l + m)!]^{1/2} P_l^m(\cos \theta) e^{im\varphi}$, $m = -l, -l + 1, \dots, l$,
 $l = 0, 1, 2, \dots$,** Spherical harmonic, 7.4.
 **$P_u^{-m, n}(\cos \theta) = [(\sin \theta)^{m-n}(1 + \cos \theta)^{u+n-m}/2^u \Gamma(m - n + 1)] {}_2F_1(-u - n, m - u;$
 $m - n + 1; [(\cos \theta - 1)/(\cos \theta + 1)])$,** Spherical function (generalized), 7.2.
 S_n, S_α Symmetric group, 1.1, 4.4.
 S_X Symmetric group on set X , 1.4.
 $Sp(m)$ Symplectic group, 5.4, 9.4.
 $a_{j_1 \dots j_n}$ Tensor components, 3.8.
 $T_1 \otimes T_2 (T^{\otimes n})$ Tensor product of group reps (n -fold), 3.5 (3.8).
 $u \otimes v$ Tensor product of vectors, 3.5.
 $V \otimes W (V^{\otimes n})$ Tensor product of vector spaces (n -fold), 3.5 (3.8).
 ${}_{2S+1}L$ Term, 9.8.
 T Tetrahedral group, 2.4.
 $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ 3- j coefficients, 7.7.
 $\text{tr } A = \sum_{i=1}^n A_{ii}$ Trace of $n \times n$ matrix A , 2.1.
 T_a Translation, 2.2.
 $T(n)$ Translation group in n -space, 2.2.
 A^t Transpose of matrix A , 2.1.
 $U(m), USp(2m), SU(m)$ Unitary classical groups, 5.4, 9.4.
 u, v, x Vectors, 2.1, 3.1.
 V, W Vector spaces, 3.1.
 V_G Volume of compact group G , 6.2.
 $V(Q)$ Volume of parallelepiped Q , 2.6.
 $\Lambda(\mathcal{H})$ Weight, 9.1.
 $S^\alpha \Lambda = \Lambda - 2(\Lambda_\alpha / \alpha_\alpha) \alpha$ Weyl reflection, 9.1.
 Z Zero matrix, 3.7.
 Z Zero operator ($Zv = \theta$), 3.7.
 θ Zero vector, 2.2.