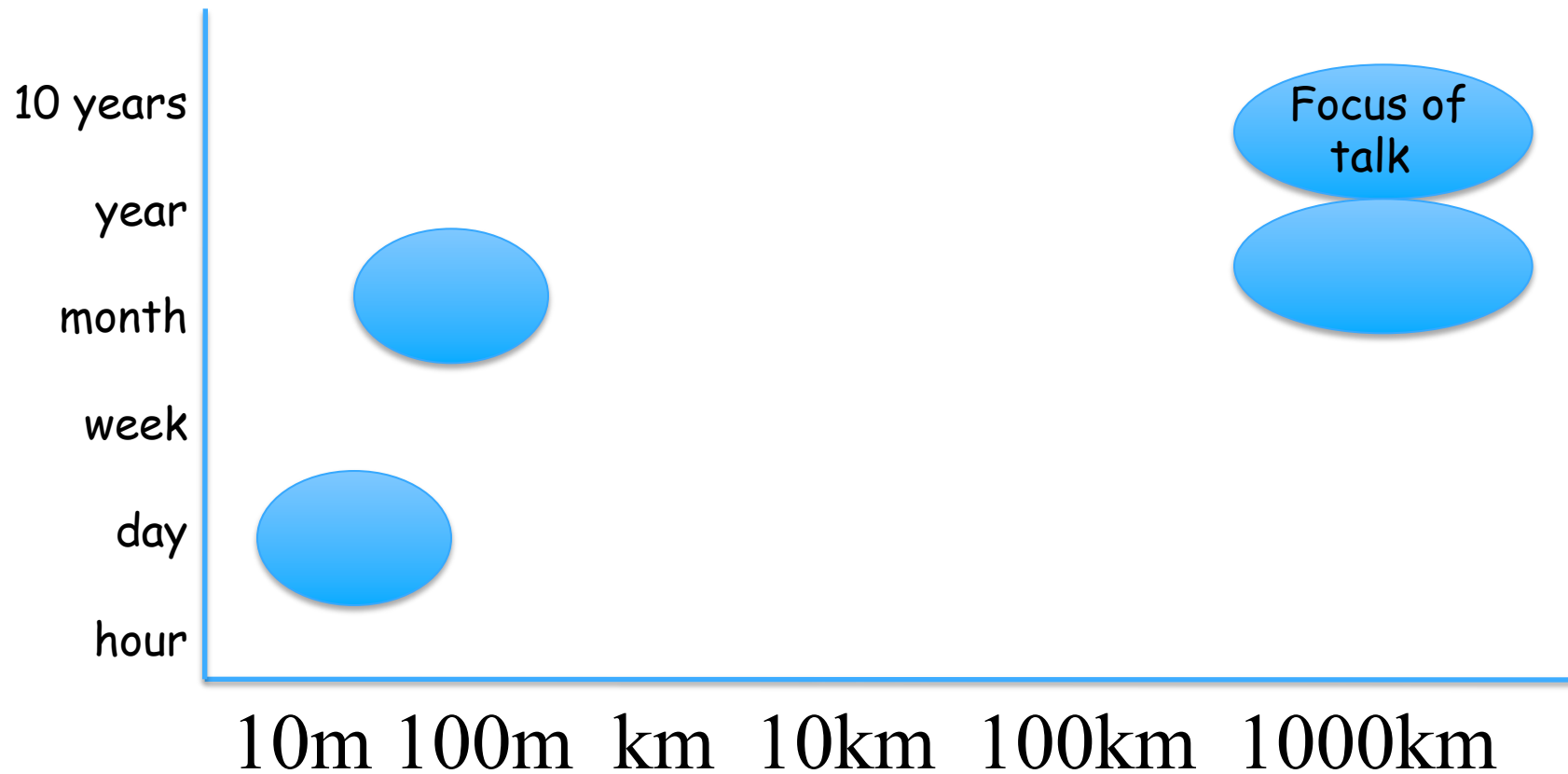


Auction Design and Sale of Wireless Spectrum

Bruce Hajek (with thank you to collaborators:
Vineet Abhishek, Walmart Labs, and
Steven R. Williams, Professor of Economics)

On what spatial and temporal scales might spectrum be traded?



Outline

1. A bit on wireless spectrum auctions in practice (US Auction 73, upcoming white space auction, UK auctions)
2. Profit sharing contracts (as in India's 3G auction) for sale of a single item and correlated private valuations
3. Core projecting auctions

1. FCC Auction 73, 2008

Auction of 700 MHz band licenses in USA
1,099 licenses offered in Auction 73.

January-March 2008 Releasing some of old TV
spectrum



All blocks in 700-800 MHz band:

Block A:12MHz	176 licenses	Reserve price \$1.8B
Block B:12MHz	734 licenses	Reserve price \$1.3B
Block D:10MHz	1 license*	Reserve price \$1.3B
Block E:6 MHz	176 licenses (unpaired spectrum)	
Reserve price \$0.9B		

(*Block D is subject to public/private partnership agreement)

Each license: -number of bidding units = upfront payment,
-min opening bid

044-A Knoxville TN, pop. 983,000, 492K bidding units
min opening bid: \$720K

068-A Champaign-Urbana, pop. 631,000, 292K bidding units,
min opening bid: \$227K

Bidding proceeds in rounds. Bidders place bids on licenses. A *provisional winner* is selected for each license from among the highest bidders.

- Ties are broken randomly.

- Carry over previous winner if no new bids.

Anonymity

The identities/owners of the bidders are released before the bidding begins. *No other info is released.*

Activity rule (“use it or lose it”). Bidders must use at least 80% of their eligibility (based on bidding units) in each round, or reduce eligibility.

Percent increases to 95% later in auction “Stage 2”

Exposure problem:

Bidder may want to acquire a bundle of licenses, but may have little interest in a partial bundle.

i.e. may want *complementary* licenses

Bidder may find it impossible to win, but get stuck with a partial package.

Early rounds offer price discovery--reduces chance of exposure.

FCC Auction 73 package bid option reduced exposure problem:

Block C : 22MHz of spectrum, broken into 12 regions:

1-6 six regions in lower 48 states

7 Alaska

8 Hawaii

Also grouped into
three packages

10 Puerto Rico, US Virgin Islands

12 Gulf of Mexico

9 Guam, Northern Mariana Islands

11 American Samoa

Q. When could the package option make a difference?

A. One strong bidder desiring nationwide C band license, but much less interested in covering only a part of nation, versus a second, very determined bidder, who wants to cover half of nation. First bidder can bid on package aggressively without fearing exposure problem.

Auction ends when no new bids offered in a round. If the sum of provisionally winning bids for all licenses in a block exceed the reserve price for block, bids become winners.

Auction 73 outcome: Bidding took 38 days for 262 rounds. 214 bidders participated, 101 bidders won 1090 licenses. AT&T gained nationwide coverage in Block B. Verizon won nationwide coverage block C. Blocks A,B,E made reserve bids, raised around \$14B. Block D (nationwide, public safety restrictions) did not make \$1.5B reserve bid.

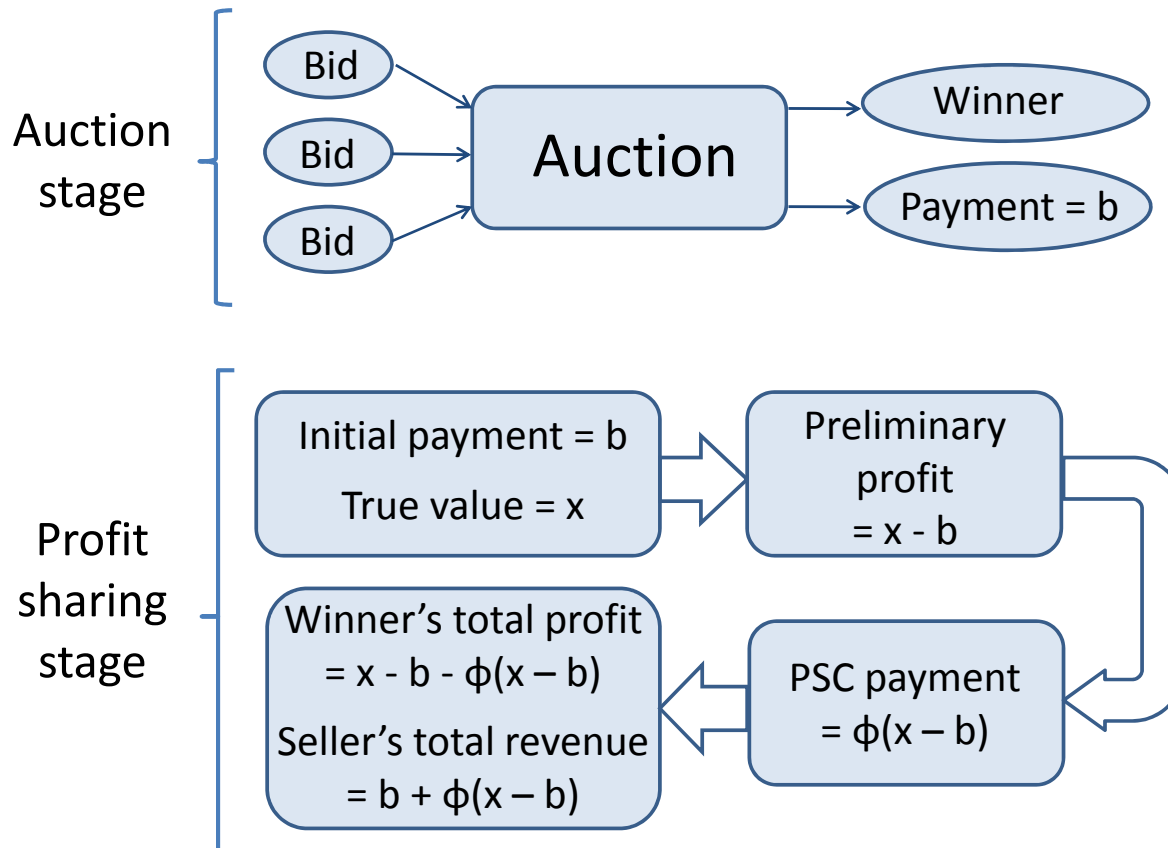
Some design choices made for Auction 73:

- What spectrum to auction and when (e.g. other blocks of 700MHz band were sold earlier)
- Granularity: Why 1099 licenses? Why groupings?
- Auction format
Choice of bidding units, minimum bids, reserve prices

Possible design goals:

- Revenue maximization? What time horizon? What revenue? Link payments to generated profits or revenues? (Being considered by Indian government.)
- Efficiency (sell to bidders who have highest value for spectrum)?
- Balance interests of incumbents vs. new entrants
- Low complexity of communication and/or computation
 - for auctioneer
 - for buyers--including burden of determining good bids

II. Auction with profit sharing contract



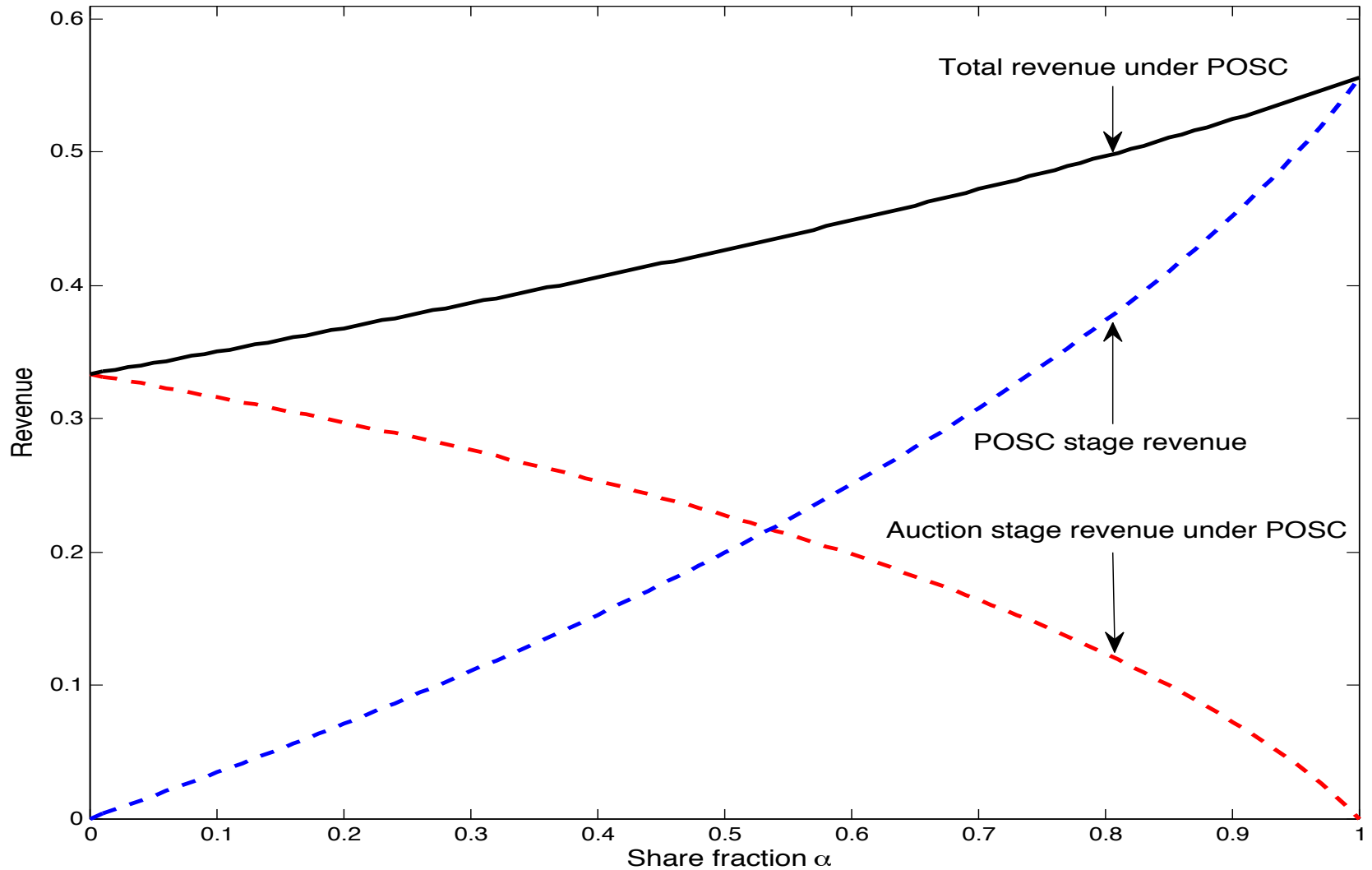
- $\phi(x - b) = \alpha[x - b]_+$ (profit only sharing)
- $\phi(x - b) = \alpha(x - b)$ (profit and loss sharing)

Example of Affiliated Private Signals

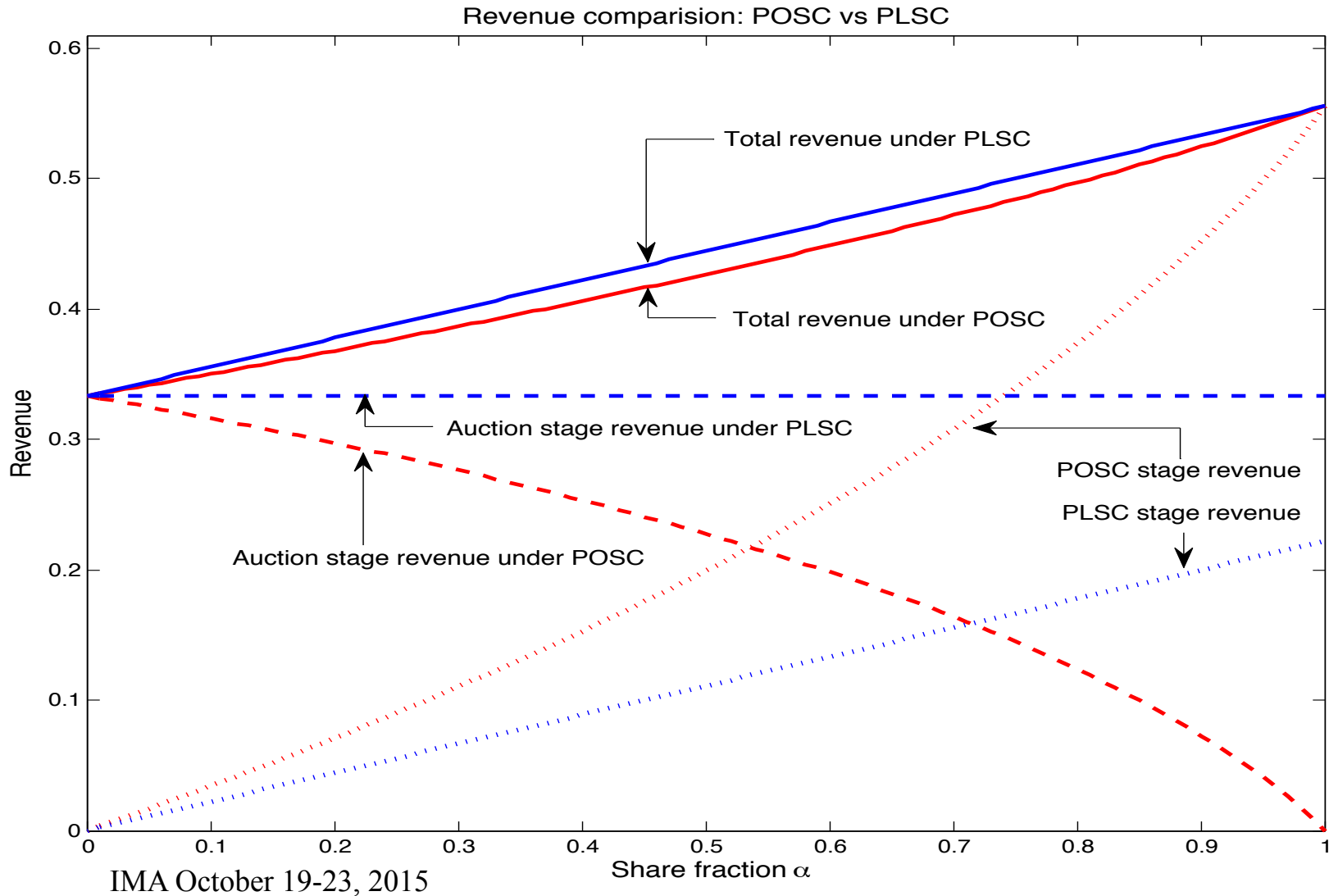
- Two risk neutral buyers.
- Values (X_1, X_2) , signals (Y_1, Y_2) .
- Signals (Y_1, Y_2) i.i.d. $\text{unif}(0, 1)$.
- $X_1 \in \{0, 1\}$, $\text{Prob}(X_1 = 1 | Y_1 = y_1, Y_2 = y_2) = (2y_1 + y_2)/3$.
 $X_2 \in \{0, 1\}$, $\text{Prob}(X_2 = 1 | Y_1 = y_1, Y_2 = y_2) = (y_1 + 2y_2)/3$.

Revenue for profit only sharing contract

Second Price Auction with POSC

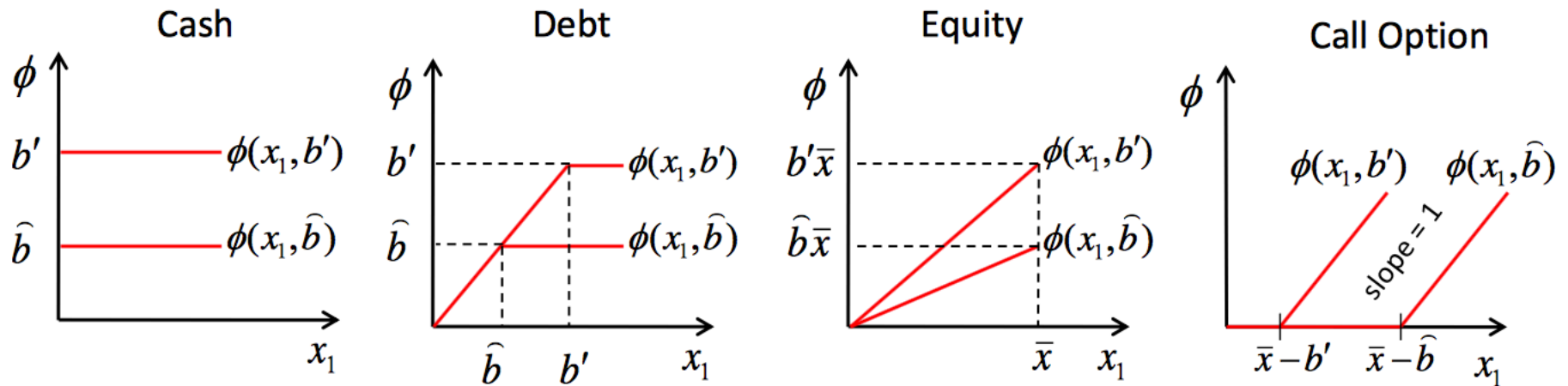


Revenue comparison: profit only sharing vs. profit and loss sharing



Demarzo, Kremer, Skrzpacz (2005) proved revenue ranking if

- (i) families of securities have steepness (single upcrossing) property
- (ii) signals and values are jointly affiliated (i.e. have log supermodular joint pdf).



Alternative (Abishek, H., Williams, 2015):

- (i) families of securities have strong steepness (monotone differences)
- (ii) value to player j is first order monotone in signal of j and highest signal of others

III. Core Selecting Auctions

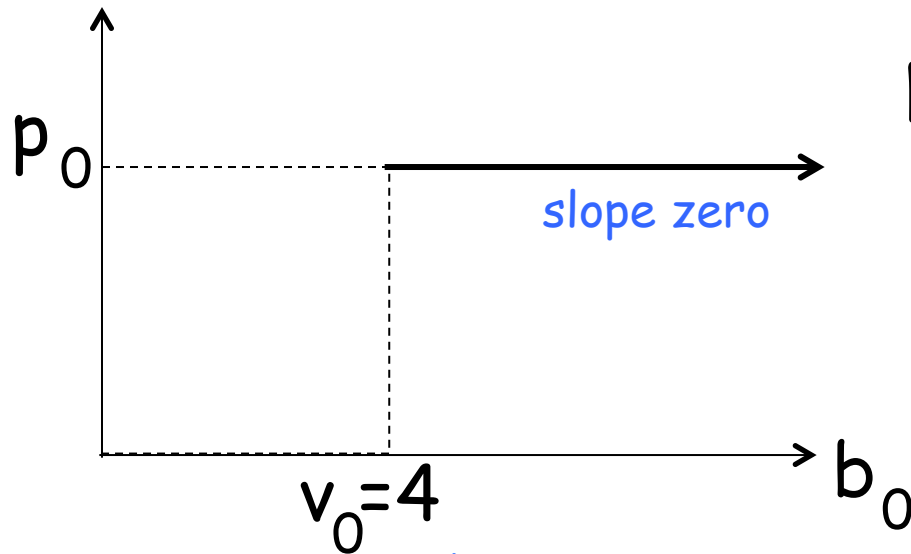
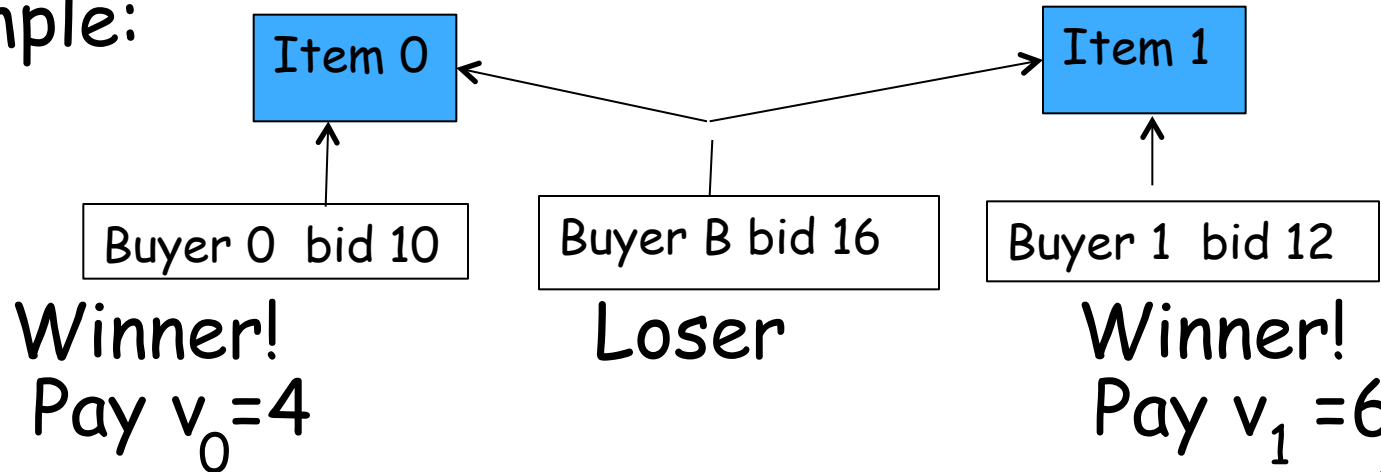
Begin with Cramton's clock package auction

- Auctioneer names prices; bidders name packages
 - Prices increase if there is excess demand
 - Process repeated until no excess demand
- Supplementary bids
 - Improve clock bids
 - Bid on other relevant packages
- Optimization to determine assignment/prices
- No exposure problem (package auction)
- Vickrey second pricing to encourage truthful bidding
- Activity rule to promote price discovery

*For details see Peter Cramton, [“Spectrum Auction Design,”](#) *Review of Industrial Organization*, 42:2, 161-190, March 2013.*

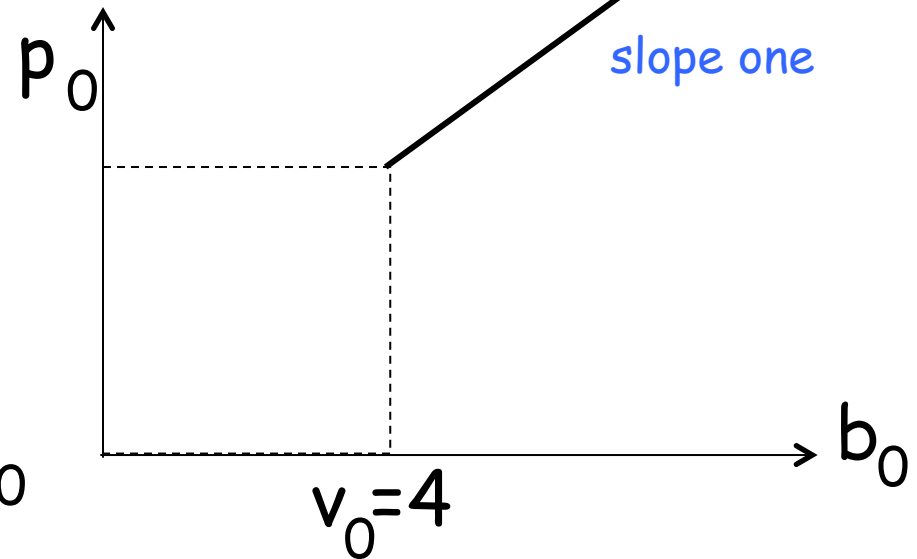
Vickrey prices incentivize truthful bidding

Example:



Vickrey 2nd price auction

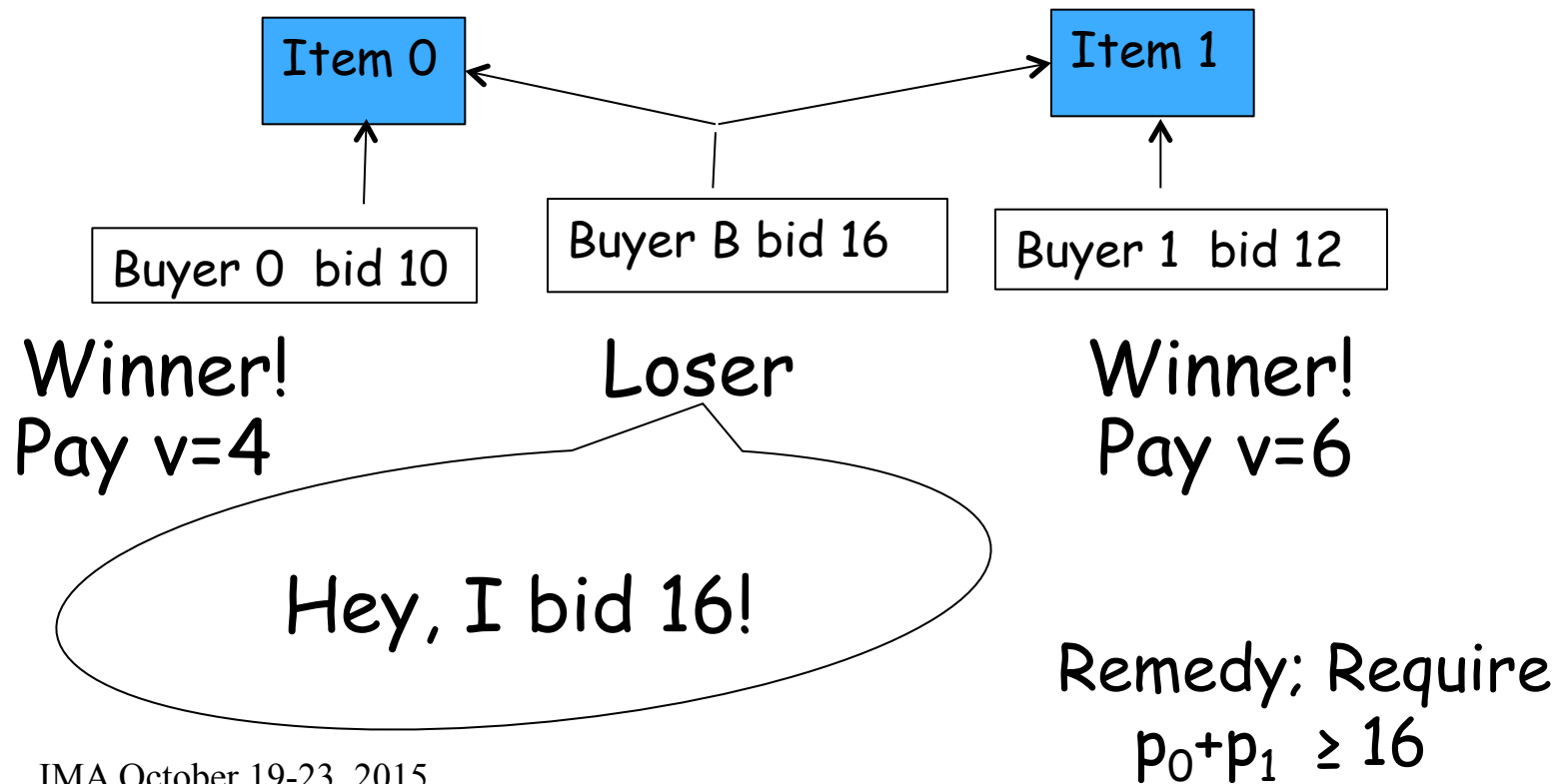
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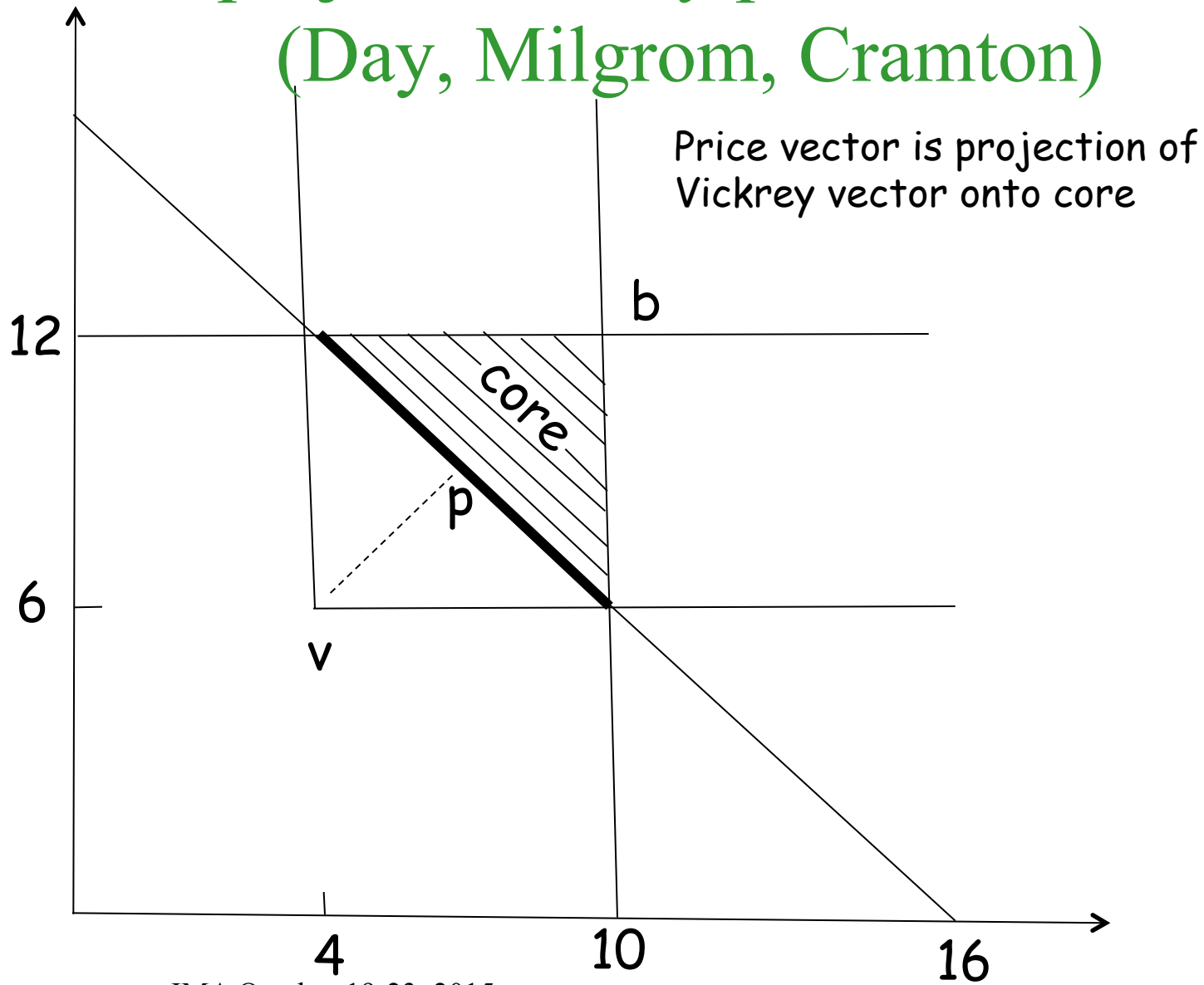
First price auction

Vickrey prices can lead to law suits

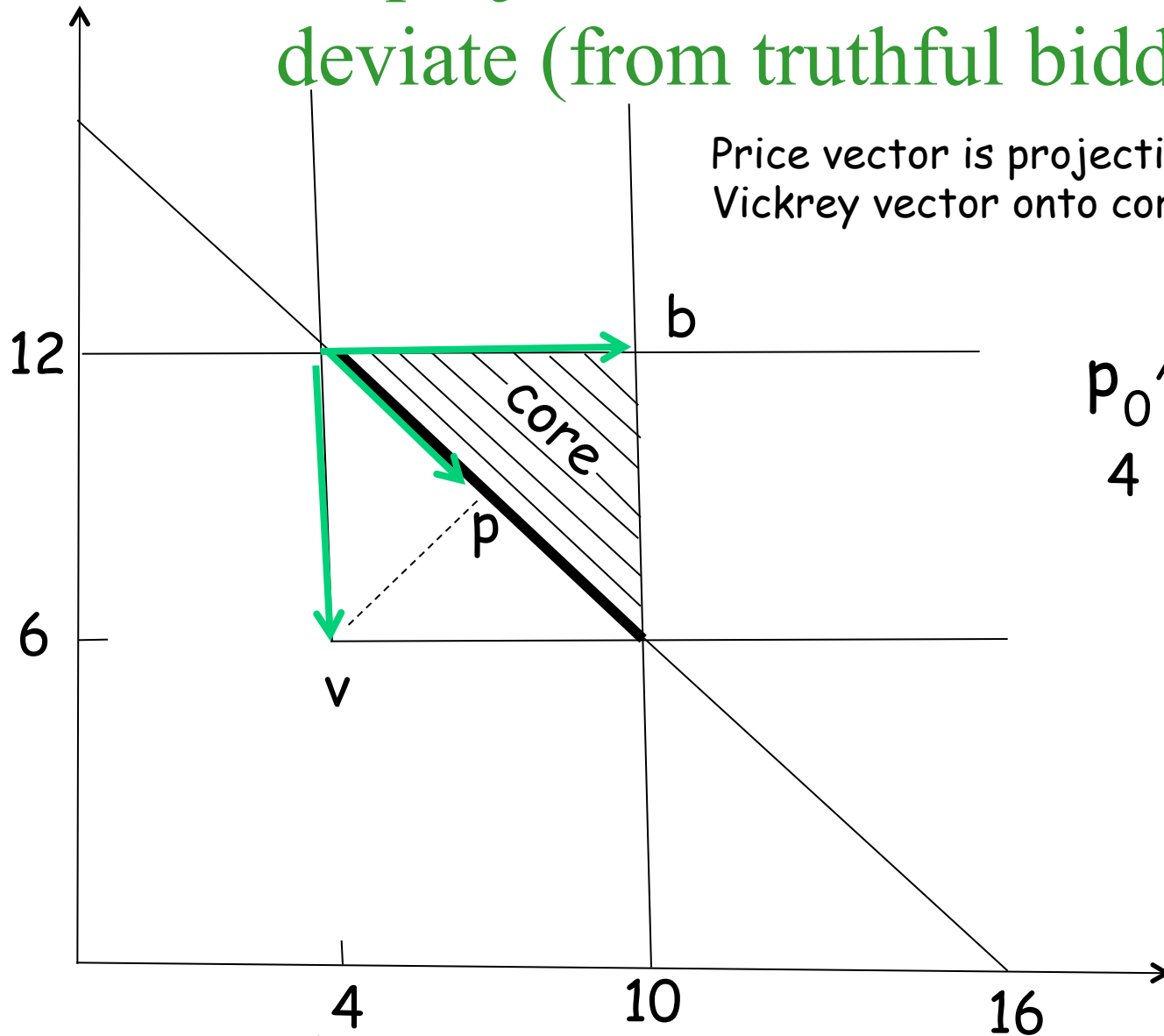
Example: Two items, three bidders.



So project Vickrey prices onto core (Day, Milgrom, Cramton)

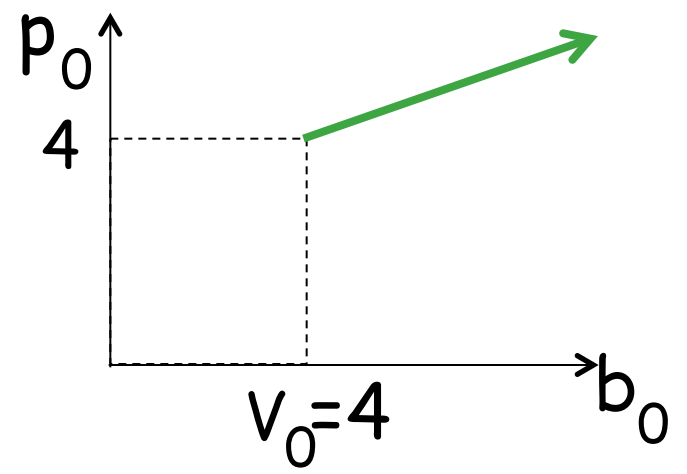


IIIa. Does projection introduce an incentive to deviate (from truthful bidding)?

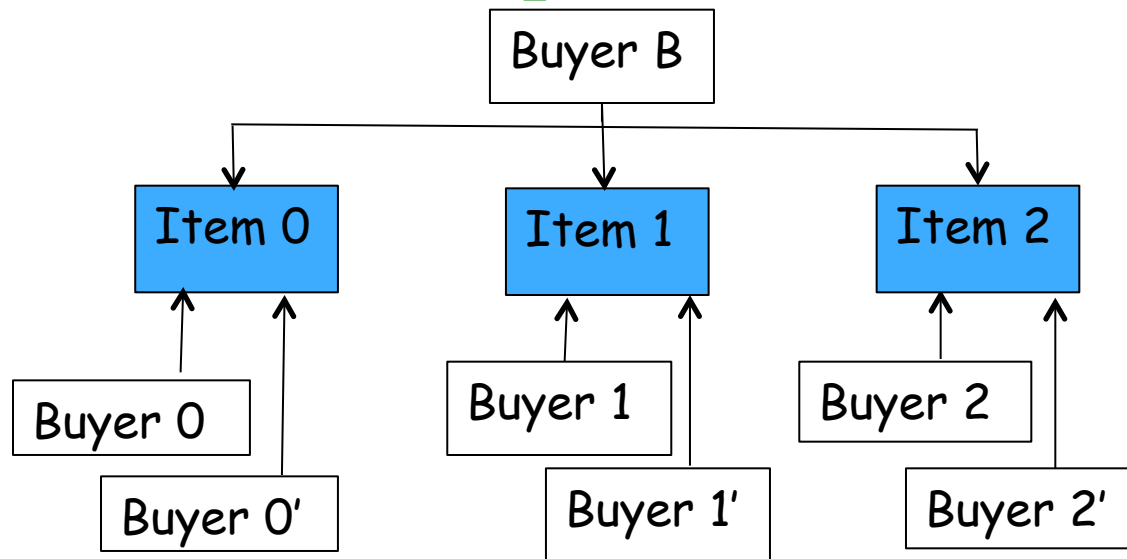


Price vector is projection of Vickrey vector onto core

Marginal Incentive to Deviate is 0.5



MID for an example with three items, 7 buyers

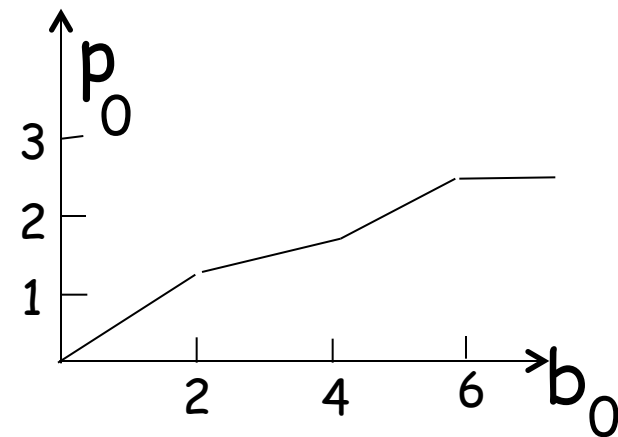


Abhishek and Hajek, Workshop on
Telecom, Economics, and Policy,
Sept. 2012

Suppose

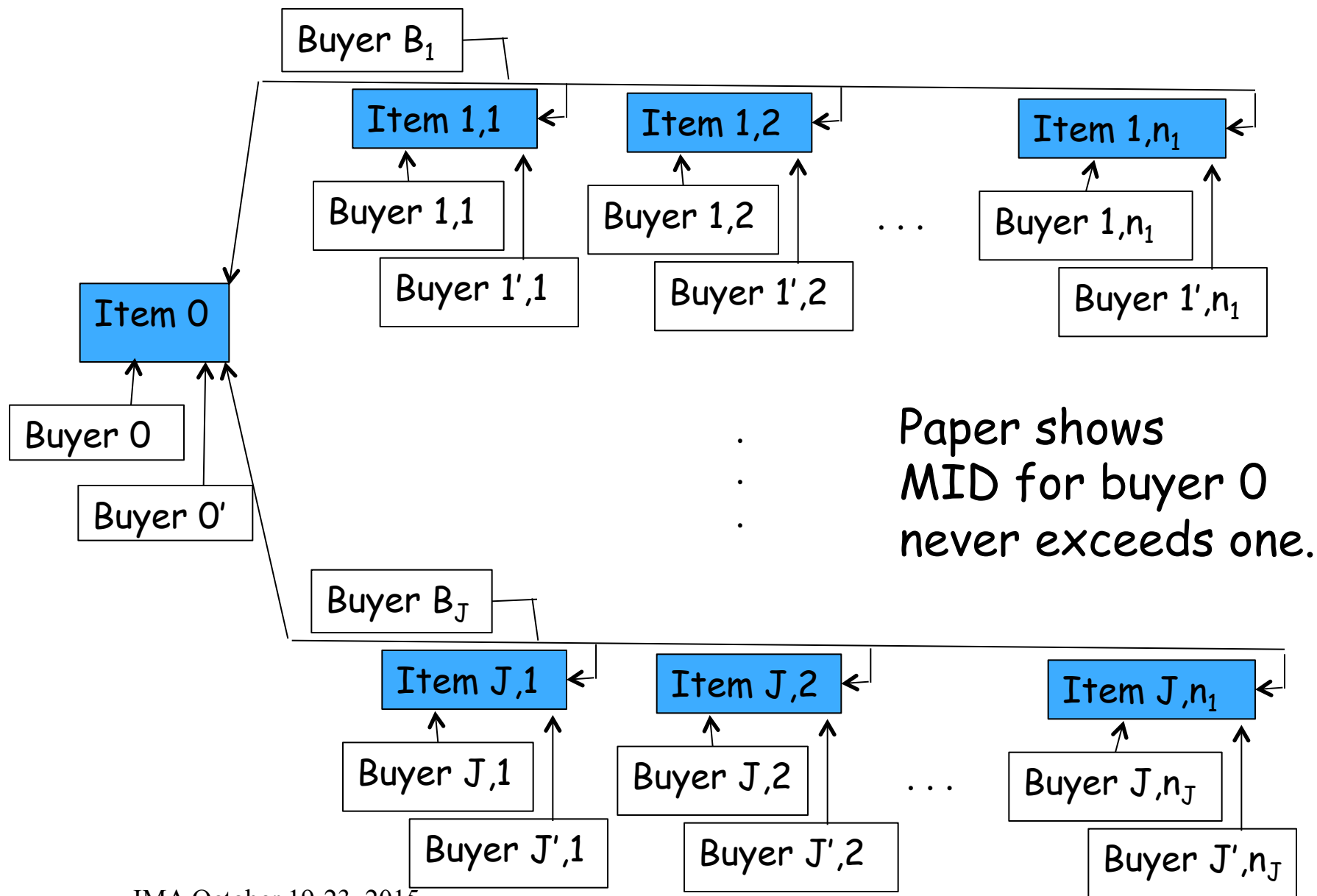
- Buyer 1 outbids buyer 1' by 6
- Buyer 2 outbids buyer 2' by 2
- Buyer B bids the sum of buyers 0',1,2
- Buyers 0,1,2 win

MID is $2/3, 1/3, 1/2, 0$,
over $[0,2],[2,4],[4,6],[6,\infty]$, respectively (not monotone)



(Both prices offset by v_0)

MID in star network setting



KKT conditions

See paper for KKT conditions. An important role is played by the Lagrange multipliers for the losing bids on bundles.

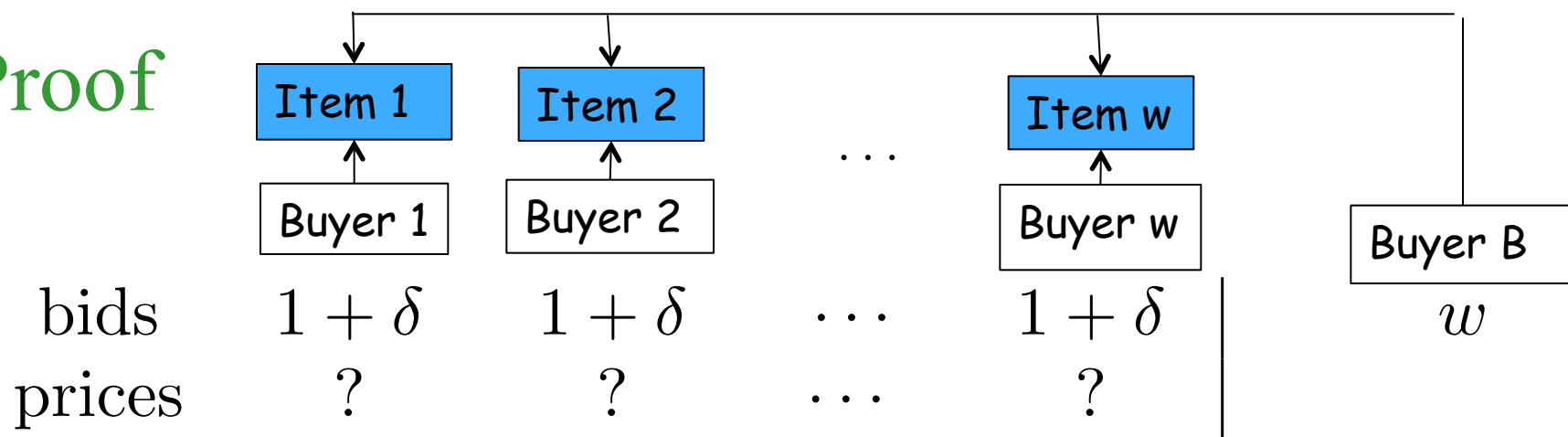
Projection onto min revenue core

- It has been suggested that the Vickrey price vector be projected onto the minimum revenue core (MRC). Our result for buyer zero of a star network, namely that MID is less than or equal to one, holds for MRC prices.

III.b Lower bound valid for any core projecting auction

Proposition For any integer $w \geq 2$, there is a scenario with w winning buyers such that $MID_{worst} \geq \frac{w-1}{w}$ for any core selecting payment rule.

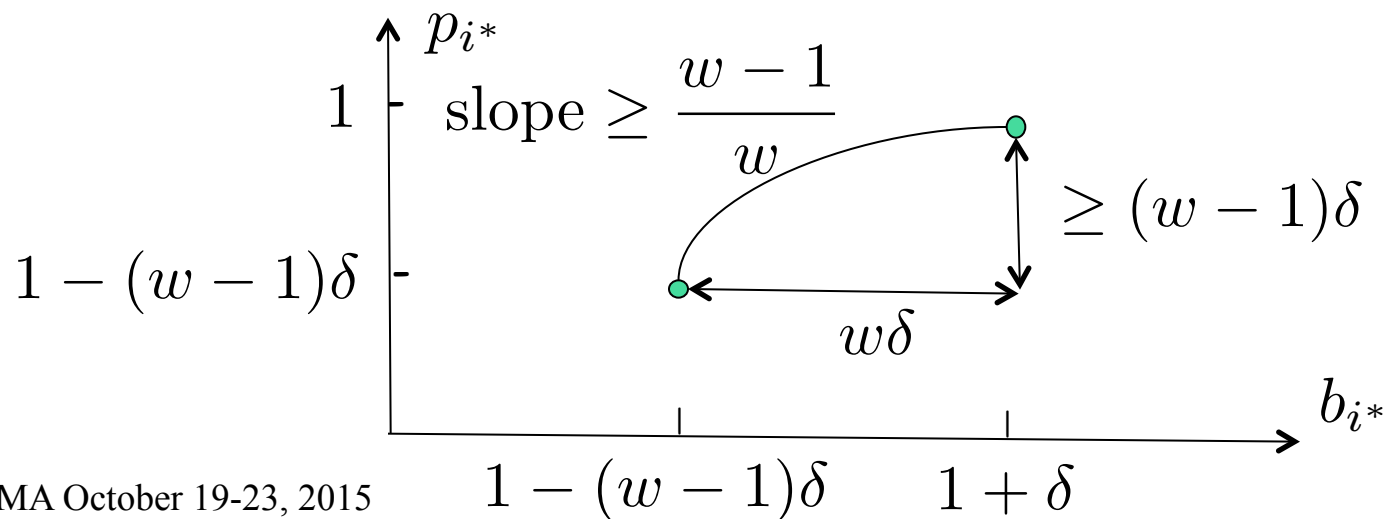
Proof



Average price must be at least one.

So $p_{i^*} \geq 1$ for some buyer i^* .

Had i^* bid $1 - (w - 1)\delta$, her price would equal bid.



Discussion

- It'd be nice to resolve whether the marginal incentive to deviate is always less than one. (Next generalization from star network would include overlapping clusters, and, finally, other buyers cutting across clusters besides buyer zero, and also buyers that are interested in more than one bundle.)
- There is an urgent need for a better understanding of the properties of auction mechanisms being proposed for upcoming sales of wireless spectrum.

Some directions for future work

- Complexity of general winner determination problem remains
- How close to incentive compatible does mechanism remain when Vickrey prices are projected onto core?
- Can affiliation of signals theory be extended from sale of single items to combinatorial auction setting?
- What is the impact of changes in the activity rules?
- What is the impact of changes in which price vector in the core is selected?

Thanks!

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