Fast moving dynamical contact lines: a capillary ballistic perspective

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work with Yi (James) Xia

Yi Xia

IMA Workshop “Dynamic contact lines . . .”
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fast moving contact lines

this talk

closed

yesterday
fast moving contact lines

slowed 170x

H₂O

\[ r \sim 1 \text{ mm} \]

\[ f \sim 61 \text{ Hz} \]

\[ a \sim 1.5 \text{g} \]

\[ Ca = \frac{\mu U_{CL}}{\sigma} \sim 10^{-3} \]

\[ Re = \frac{\rho U_{CL} r}{\mu} \sim 160 \]

capillary ballistics
– inertia vs surface tension

• fluid inertia vs Young-Laplace capillarity
• ideal fluid motions – slippery boundaries

Borkar et al 1991; Lyubimov 2001, 06; Fayzrakhmanova et al 2009; etc

Carlson et al 2012

physically-based CL kinetics for \textit{fast} motions?
-- Cox 1998

Rayleigh half-drop: \( Re = \infty, Ca = 0 \)
exact soln -- if CA=90, no CA hysteresis.
frequency scan – ballistics

\[ \frac{\hat{Y}}{\hat{X}} \]

low damping

high damping

\[ \omega / \omega_n \]

damping ratio, \( \zeta = \frac{c}{2\sqrt{km}} \)
frequency scan

\[ \frac{Y}{X} \]

<table>
<thead>
<tr>
<th>Driving frequency, ( f ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

[2,0] [4,0]

67

JFM, Bostwick & phs 2014; Chang et al 2015
CL motions – measurement
CL motion

Dussan 1979

water on low hysteresis Si wafer
## Parameter Details

<table>
<thead>
<tr>
<th>System</th>
<th>Dimensional parameters</th>
<th>Dimensionless parameters (symbol and definition)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$a$ (ms$^{-2}$)</td>
</tr>
<tr>
<td>M00</td>
<td>61</td>
<td>15</td>
</tr>
<tr>
<td>F00</td>
<td>66</td>
<td>6.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (ms)</th>
<th>Length (mm)</th>
<th>Velocity (mm s$^{-1}$)</th>
<th>Acceleration (m s$^{-2}$)</th>
<th>Angle (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\omega$</td>
<td>$\tau^*$</td>
<td>$D$</td>
<td>$A^*$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>2.6</td>
<td>17</td>
<td>2.7</td>
<td>0.10</td>
<td>0.051</td>
</tr>
</tbody>
</table>

**TABLE 1.** Scales: forcing, $\omega^{-1} \equiv (2\pi f)^{-1}$; capillary, $\tau \equiv \sqrt{\rho V/\sigma}$; volume-based length, $D \equiv V^{1/3}$; typical forcing amplitude, $A^*$; boundary layer thickness, $\delta \equiv \sqrt{v/\omega}$; forcing speed, $A^*\omega$, CL speed, $U_{CL}$; forcing acceleration $A^*\omega^2$; maximum angle-deviation, $\Delta\alpha^*$; asterisk (*) denotes value fixed throughout the paper, a typical value used for scaling.
more details – systems

<table>
<thead>
<tr>
<th>Designation</th>
<th>Substrate</th>
<th>Surface treatment</th>
<th>Water $\alpha_A$ (°)</th>
<th>Water $\alpha_R$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mxx (Mobile)</td>
<td>Silicon</td>
<td>Trimethylsiloxy terminated PDMS</td>
<td>102</td>
<td>100</td>
</tr>
<tr>
<td>Fxx (Fluorinated)</td>
<td>Silicon wafer</td>
<td>Fluorinated trichlorosilane</td>
<td>120</td>
<td>92</td>
</tr>
</tbody>
</table>
‘mobility’ – say what?

\[ F = 6\pi \mu R \ U \] recall Stokes drag

uncompensated Young force

\[ F = 2\pi R \sigma \ [\cos \bar{\alpha} - \cos(\bar{\alpha} + \Delta \alpha)] = \left( \frac{1}{M^*} \right) U_{CL} \]

\[ F \approx 2\pi R \sigma \ \sin(\bar{\alpha}) \bigg|_{\pi/2} \Delta \alpha \approx (2\pi R \sigma) \ \Delta \alpha \]

\[ (2\pi R \sigma) M^* \ \Delta \alpha = U_{CL} \]

\[ M \ \Delta \alpha = U_{CL} \]

mobility
cyclic CL motion

wing advancing

wing receding
cyclic motion
cyclic diagram & mobility

\[ M \Delta \alpha = U_{CL} \]

\[ M \sim 3.5 \text{ cm/s-10 degree} \]
### mobility – other systems

<table>
<thead>
<tr>
<th>System</th>
<th>$a$ (m s$^{-2}$)</th>
<th>$V$ (µL)</th>
<th>$\Lambda/(\Delta \alpha^* \tau^<em>/A^</em>)$</th>
<th>$M/(A^<em>/\Delta \alpha^</em> \tau^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M00</td>
<td>14.3</td>
<td>20</td>
<td>0.14</td>
<td>7.1</td>
</tr>
<tr>
<td>F00</td>
<td>13.3</td>
<td>20</td>
<td>0.28</td>
<td>3.6</td>
</tr>
<tr>
<td>M40</td>
<td>12.8</td>
<td>20</td>
<td>0.34</td>
<td>2.9</td>
</tr>
<tr>
<td>M60</td>
<td>13.0</td>
<td>20</td>
<td>0.45</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**Table 5.** Measured $\Lambda$ and $M$ for Mxx and F00 systems. $\Delta \alpha^* \tau^*/A^* = 17$ s° cm$^{-1}$. 

introduce mappings to *unfold* TD – ‘cyclic diagram’

• measure *mobility*, purely experimentally \( M \Delta \alpha = U_{CL} \)

• mobility characterizes CL kinetics – *far from stick-slip*

\[ M \sim 3.5 \text{ cm/s-10 degree} \]

Xia & Steen, JFM 2018
capillary ballistic perspective

• fluid inertia balances Young-Laplace capillarity

• ideal fluid motions – slippery boundaries

• kinetics of CL – contact-line drag

“The existence of tangential stresses and the impossibility of slip constitute, then, the differences between a real fluid and a theoretically ideal one. In this book we are concerned with the motion of those fluids whose viscosity is small: the phenomena in such motions sometimes approximately agree, sometimes violently disagree, with the predictions of ideal fluid theory...” Sydney Goldstein, “Modern Developments in Fluid Dynamics” volume 1, preface, 1938.