Computational Radar Imaging

Müjdat Çetin

Associate Professor, Department of Electrical and Computer Engineering
Interim Director, Goergen Institute for Data Science

University of Rochester, Rochester, NY

Radar Imaging Basics

- All-weather
- Day and night operation
- Superposition of response from scatterers – tomographic measurements

- Synthetic aperture radar (SAR)
- Computational imaging problem: Obtain a spatial map of reflectivity from radar returns
Outline

• Sparsity-driven radar imaging
  – Point-enhanced and region-enhanced imaging
  – ADMM & proximal operators in the case of complex-valued fields
  – Wide-angle imaging and anisotropy characterization
  – Model errors and autofocusing
  – Moving-object imaging

• Machine learning for radar imaging
  – Dictionary learning
  – Deep learning-based priors
Initial motivation for our work (*circa 1999*)

Some challenges for automatic decision-making from SAR images:

- Accurate localization of **dominant scatterers**
  - Limited resolution
  - Clutter and artifact energy

- **Region separability**
  - Speckle
  - Object boundaries

- Limited or sparse apertures

* This slide was found in an archaeological excavation site and is believed to be ~2000 deep-learning-years old.
Maximum Entropy and the Nearly Black Object

By DAVID L. DONOHO, IAIN M. JOHNSTONE†, JEFFREY C. HOCH and ALAN S. STERN

University of California, Berkeley, USA
Stanford University, USA
Rowland Institute for Science, Cambridge, USA

[Read before The Royal Statistical Society at a meeting organized by the Research Section on Wednesday, April 17th, 1991, Dr F. Critchley in the Chair]

SUMMARY
Maximum entropy (ME) inversion is a non-linear inversion technique for inverse problems where the object to be recovered is known to be positive. It has been applied in areas ranging from radio astronomy to various forms of spectroscopy, sometimes with dramatic success. In some cases, ME has attained an order of magnitude finer resolution and/or an order of magnitude smaller noise level than that obtainable by standard linear methods. The dramatic successes all seem to occur in cases where the object to be recovered is ‘nearly black’: essentially zero in the vast majority of samples. We show that near-blackness is required, both for signal-to-noise enhancements and for superresolution. However, other methods—in particular, minimum $l_1$-norm reconstruction—may exploit near-blackness to an even greater extent.

Keywords: DIFFRACTION-LIMITED IMAGING; INVERSE PROBLEMS; MAGNETIC RESONANCE SPECTROSCOPY; MINIMAX DECISION THEORY; NON-LINEAR RECOVERY; POSITIVITY CONSTRAINTS; SUPERRESOLUTION
SAR Ground-plane Geometry

- Scalar 2-D complex-valued reflectivity field \( f(x, y) \)
- Transmitted chirp signal: \( s(t) = \Re \left[ e^{j(\omega_0 t + \alpha t^2)} \right] , \ |t| \leq \frac{T_p}{2} \)
- Received, demodulated return from circular patch:

\[
r_{\theta}(t) = \int_{|u| \leq L} q_{\theta}(u) \exp \left\{ -j \frac{2}{c} \left[ \omega_0 + 2\alpha \left( t - \frac{2R}{c} \right) \right] u \right\} du
\]
SAR Observation Model

- Observations are related to projections of the field:

\[
r_\theta(t) = \int_{|u| \leq L} g_\theta(u) \exp \left\{ -j \frac{2}{c} \left[ \omega_0 + 2\alpha \left( t - \frac{2R}{c} \right) \right] u \right\} du
\]

- SAR observations are band-limited slices from the 2-D Fourier transform of the reflectivity field:

\[
r_\theta(t) = \int \int_{x^2 + y^2 \leq L^2} f(x, y) \exp \left\{ -j\Omega(t) (x \cos \theta + y \sin \theta) \right\} \, dx \, dy
\]

\[
= F \left[ \Omega(t) \cos \theta, \Omega(t) \sin \theta \right]
\]

- Discrete tomographic SAR observation model:
  (combining all measurements)
Conventional Image Formation

- Given SAR returns, create an estimate of the reflectivity field $f$

Polar format algorithm:
- Each pulse gives slice of 2-D Fourier transform of field
- Polar to rectangular resampling
- 2-D inverse DFT
Sparsity-Driven Radar Imaging – basic version

\[ J(f) = \|y - Af\|_2^2 + \lambda \|Lf\|_p^p \]

- Bayesian interpretation: MAP estimation problem with heavy-tailed priors \( p(f \mid y) \propto p(y \mid f) p(f) \)
- Complex-valued data and image
- Magnitude of complex-valued field admits sparse representation
- No informative prior on reflectivity phase
- Typical choices for L:
  - identity (point-enhanced imaging)
  - gradient (region-enhanced imaging)
- Here focus on ADMM-based solutions
Setting up with a generic regularizer

• Discretized SAR observation model:
  \[ y = Af + n \]

• Retrieve \( f \) using a regularized cost function:
  \[ \hat{f} = \arg\min_f \{ D(f) + \lambda R(|f|) \} \]
  where \( D(f) = \|y - Af\|_2^2 \) and \( R(|f|) \) is the regularizer

• Will describe two versions of ADMM
Taking into account the complex-valued nature

- Rewrite \( f = \Theta |f| \) where \( \Theta \) is a diagonal matrix containing the phase of \( f \) in the form \( e^{j\phi(f)} \)

- Cost function becomes:

\[
\{ |\hat{f}|, \hat{\Theta} \} = \arg\min_{|f|, \Theta} \|y - A\Theta|f|\|^2 + \lambda R(|f|)
\]
Variable Splitting and ADMM

- Introduce an auxiliary variable with a constraint:

\[
\{ |\hat{f}|, \hat{\Theta}, \hat{h} \} = \arg \min_{|f|,\Theta,h} \|y - A\Theta f\|^2_2 + \lambda \mathcal{R}(h)
\]

\[
s. t. |f| - h = 0
\]

- Augmented Lagrangian (in scaled form):

\[
\{ |\hat{f}|, \hat{\Theta}, \hat{h}, \hat{u} \} = \arg \min_{|f|,\Theta,h,u} \|y - A\Theta f\|^2_2 + \lambda \mathcal{R}(h)
\]

\[
+ \frac{\rho}{2} \| |f| - h + u \|^2_2 + \frac{\rho}{2} \| u \|^2_2
\]
Variables involved - details

• Let $\theta \in \mathbb{C}^{N \times 1}$ be a vector containing the diagonal elements of the phase matrix $\Theta$

• Invoke the constraint that the magnitudes of the elements of $\theta$ should be 1, since they contain phases in the form $e^{j\phi(f)}$

• Let $B$ be a matrix whose diagonal elements contain the reflectivity magnitudes

• Let $\tilde{f} = \hat{h} - u$ and $\tilde{h} = |\hat{f}| + u$
ADMM for Complex-valued Imaging

- Each iteration of the ADMM algorithm performs the following steps enabling the use of a Plug-and-Play (PnP) prior approach:

\[
\hat{\theta} = \arg\min_{\theta} \| y - A B \theta \|^2_2 + \lambda_\theta \sum_{i=1}^{N} (|\theta_i| - 1)^2
\]

\[
|\hat{f}| = \arg\min_{|f|} \| y - A \Theta |f| \|^2_2 + \frac{\rho}{2} \| |f| - \tilde{f} \|^2_2
\]

\[
\hat{h} = \arg\min_{h} \lambda \Re(h) + \frac{\rho}{2} \| \hat{h} - h \|^2_2 \quad \text{All real-valued}
\]

\[
\hat{u} = u + |\hat{f}| - \hat{h}
\]

where \( \lambda_\theta \) is a hyperparameter
Alternative way to set up ADMM

• Introduce an auxiliary variable with a constraint:

\[
\{\hat{f}, \hat{h}\} = \arg\min_{f, h} \|y - Af\|^2_2 + \lambda \mathcal{R}(|h|)
\]

s.t. \( f - h = 0 \)

• Augmented Lagrangian (in scaled form):

\[
\{\hat{f}, \hat{h}, \hat{u}\} = \arg\min_{f, h, u} \|y - Af\|^2_2 + \lambda \mathcal{R}(|h|)
\]

\[
+ \frac{\rho}{2} \|f - h + u\|^2_2 + \frac{\rho}{2} \|u\|^2_2
\]

• Let \( \tilde{f} = \hat{h} - u \) and \( \tilde{h} = \hat{f} + u \)
ADMM-II for Complex-valued Imaging

- Each iteration of the ADMM algorithm performs the following steps enabling the use of a Plug-and-Play (PnP) prior approach:

\[
\hat{f} = \arg \min_f \|y - Af\|^2 + \frac{\rho}{2} \|f - \tilde{f}\|^2
\]

\[
\hat{h} = \arg \min_h \lambda \Re(|h|) + \frac{\rho}{2} \|\tilde{h} - h\|^2
\]

\[
\hat{u} = u + \hat{f} - \hat{h}
\]

where \(\lambda_0\) is a hyperparameter.
Proximal Mappings for Complex-Valued Problems

- Consider total variation (TV) regularization.
- Proximal mapping for standard TV can be computed by efficient algorithms
  \[
  \prox_{TV}(\mathbf{v}) = \arg \min_{\mathbf{x}} TV(\mathbf{x}) + \frac{\mu}{2} \|\mathbf{x} - \mathbf{v}\|_2^2
  \]
- We want to penalize TV of the magnitude of a complex-valued field
  \[
  TV(|\mathbf{x}|) = \sum_{i,j} \nabla |\mathbf{x}|[i,j].
  \]
  \[
  (D_1|\mathbf{x}|) = |\mathbf{x}[i+1,j]| - |\mathbf{x}[i,j]|,
  \]
  \[
  (D_2|\mathbf{x}|) = |\mathbf{x}[i,j+1]| - |\mathbf{x}[i,j]|.
  \]
- Proximal mapping operator for TV-magnitude:
  \[
  \prox_{TV(|\cdot|)}(\mathbf{v})[i,j] = \prox_{TV(|\mathbf{v}|)}[i,j] \cdot \exp\{j \angle (\mathbf{v}[i,j])\}
  \]
- Similarly, proximal mapping operator of a linear transformation (Wavelet trf., DCT,...) of the magnitude of the complex field:
  \[
  \prox_{\|W|x\|_1}(\mathbf{v}) = \prox_{\|Wx\|_1}(|\mathbf{v}|) \cdot \exp\{j \angle (\mathbf{v})\}
Proximal Mapping for $l_p$-norm-based Regularizers

- Proximal mapping for $l_p$-norm

$$\text{prox}_{\|\cdot\|_p}(v) = \arg \min_x \|x\|_p^p + \frac{\mu}{2}\|x - v\|_2^2$$

- Can be computed through a series of iteratively reweighted soft thresholding operations:

$$\text{prox}_{\|\cdot\|_p}(v)[i] = \frac{1}{w[i]} \text{soft}(w[i] \times v[i], p/\mu)$$

$$w[i] = (v[i] + \beta)^{(1-p)}$$
Sparsity-Driven SAR Imaging Results

Conventional

Point-enhanced  Region-enhanced  Wavelet dictionary
Sparsity-Driven SAR Imaging Results

Conventional

Sparsity-driven, ADMM-based
Non-Traditional Apertures: Wide-Angle Data

- Irregular PSF
- Anisotropic scattering
  - Conventional imaging produces inaccurate reflectivities
  - Conventional imaging does not characterize aspect dependence
Anisotropic Scattering in SAR

• Much reflection from one angle
Anisotropic Scattering in SAR

• Little reflection from another angle
Composite Image Formation

- Many scatterers won’t persist over wide angles
- Form a composite wide-angle image from narrow-angle subaperture images:

\[
\hat{f}_{ij} = \arg \max_k \hat{f}_{ij}^k
\]

- Preservation of anisotropic scatterers with short persistence
- Partial characterization of aspect dependence
Wide-angle SAR Imaging

- Angular range = 110°
- Composite images
Wide-Angle SAR Imaging with Frequency-Band Omissions

- 70% of the band available

Reference image

Conventional

Sparsity-driven
Visualization of aspect dependence in wide-angle imaging
Joint Imaging and Anisotropy Characterization

- Model for data collected from $P$ isotropic scatterers:

$$r(\omega, \theta) = \sum_{m=1}^{P} f(x_m, y_m) \exp \left\{ -j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}$$

- Image formation: Recovering $f(x,y)$

- When we consider anisotropy:

$$r(\omega, \theta) = \sum_{m=1}^{P} f(x_m, y_m, \theta) \exp \left\{ -j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}$$

- Joint imaging and anisotropy characterization: Recovering $f(x,y,\theta)$

- The problem becomes much more underdetermined!
Modeling Angular Anisotropy

• Model $f(x,y,\theta)$ by the dictionary $\{b_1(\theta), b_2(\theta), \ldots, b_M(\theta)\}$

$$r(\omega, \theta) = \sum_{m=1}^{P} f(x_m, y_m, \theta) \exp \left\{ -j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}$$

$$r(\omega, \theta) = \sum_{m=1}^{P} \sum_{i=1}^{M} a_{m,i} b_i(x_m, y_m, \theta) \exp \left\{ -j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}$$

• Problem: determine unknown coefficients $a_{m,i}$
• $b_i(\theta)$: atoms of the angular response dictionary
• Construct an overcomplete dictionary to allow sparse representation of typical angular response patterns.
Sample Result of an Analysis-based Formulation for Anisotropy Characterization

Aspect dependent scattering behavior in indicated sub-area
Joint Imaging and Model Error Correction: Sparsity-Driven Autofocus (SDA)

• SAR observation model may not be known perfectly, due to, e.g., uncertainties in platform location or atmospheric turbulence
• This leads to phase errors in observed data
• Proposed approach: joint optimization over the reflectivities and model parameters:

\[ J(f, \phi) = \|y - A(\phi)f\|_2^2 + \lambda \|f\|_1 \]

• Solution through coordinate descent or ADMM
  – Closed-form solution for the phase error estimation step
Sparsity-Driven Autofocus (SDA)

Conventional imaging

Without phase errors

Sparsity-driven imaging

Random, uniform phase error

1D Phase error uniform. dist. in $\left[ -\frac{\pi}{2}, +\frac{\pi}{2} \right]$
Synthesis-based SDA

Reference conventional image without phase errors

Conventional image with phase errors

Proposed approach with phase errors

Wavelet dictionary
Moving-Target Imaging

• SAR platform position uncertainties cause space-invariant defocusing of the reconstructed image, i.e., the amount of defocusing is the same for all points in the scene.

• Motion of a target in the scene can also be modeled as a phase error over the phase history data corresponding to a stationary scene.

• Moving targets in the scene cause artifacts including defocusing around the spatial neighborhood of the target in the scene.
  → Space-variant defocusing
  → Need to keep an account of the contributions from each spatial location to the phase error at each aperture position
Sparsity-Driven Moving-Target Imaging

\[ \arg \min_{f, \beta} J(f, \beta) = \arg \min_{f, \beta} \| y - A(\phi) f \|_2^2 + \lambda_1 \| f \|_1 + \lambda_2 \| \beta - 1 \|_1 \]

subject to \( |\beta(i)| = 1 \quad \forall i \)

\[ \beta_m = [e^{j\phi_1(m)}, e^{j\phi_2(m)}, \ldots, e^{j\phi_t(m)}]^T \]

• Involves sparsity constraints both on the reflectivity field and on the motion field.

• Have also developed a more efficient and potentially robust version based on constructing ROIs and performing space-invariant focusing within them.
Moving-Target Imaging

Conventional imaging

Sparsity-driven imaging

Proposed method
Moving Target Imaging by Low-rank and Sparse Decomposition

\[ D = \begin{pmatrix} d^1 & d^2 & d^3 & d^4 \\ \cdots & \cdots & \cdots & \cdots \\ d^{Q-1} & d^Q \end{pmatrix} \rightarrow SLRSD \]

\[
\arg \min_{D,B,S,\Theta} \| g - C\Theta R^{-1}(D) \|_2^2 + \lambda_b \| B \|_* + \lambda_s \| S \|_1 \\
\text{s.t. } D = B + S, \quad |\Theta_{i,i}| = 1 \quad \forall i,
\]
Dictionary Learning for Sparsity-Driven SAR Imaging

Training Images

K-SVD

Learned dictionary

Conventional Learning-based
Deep Learning-based Priors for SAR Imaging

• A new SAR image reconstruction framework utilizing Plug-and-Play (PnP) priors
  – Optimization-based reconstruction - regularized inversion,
  – Decoupling the data and the prior through ADMM
  – Deep learning-based prior
Variable Splitting and ADMM

- Each iteration of the ADMM algorithm performs the following steps enabling the use of a Plug-and-Play (PnP) prior approach:

\[
\hat{\theta} = \arg\min_{\theta} \|y - AB\theta\|^2_2 + \lambda \sum_{i=1}^{N} (|\theta_i| - 1)^2
\]

\[
|\hat{f}| = \arg\min_{|f|} \|y - A\Theta|f|\|^2_2 + \frac{\rho}{2} \| |f| - \tilde{f} \|^2_2
\]

\[
\hat{h} = \arg\min_{h} \lambda \Re(h) + \frac{\rho}{2} \| \tilde{h} - h \|^2_2
\]

\[
\hat{u} = u + |\hat{f}| - \hat{h}
\]

where \(\lambda_\theta\) is a hyperparameter.
Convoluational Neural Network (CNN)-based Prior

- Architecture: modified version of the network used in [1]
- 20 convolutional modules
- First and even numbered modules: 64 3×3 filters with padding 1, stride 1
- Remaining modules: 64 5×5 filters with padding 2, stride 1
- Each module has batch normalization and ReLU layers

Training the CNN - Synthetic Scenes

Using 64×64 ground truth images for CNN training:

• Add random phase to training images and obtain the phase histories.

• Apply complex-valued additive noise to the phase histories.
  – Magnitude uniformly distributed over [0, $\sigma_Y$], where $\sigma_Y$ is the standard deviation of the magnitude of the phase history data
  – Phase uniformly distributed over $[-\pi, \pi]$

• Perform conventional reconstructions.

• Extract 16×16 overlapping patches from these conventional images and their corresponding ground truths, to construct input-output pairs.

• Augment the pairs of images through rotation by $[90^\circ, 180^\circ, 270^\circ]$.

• Train the network using these augmented pairs of images, with image reconstructed from noisy data as input and ground truth image as output.
Synthetic Data Experiments -- Training Set
Test Set

- 5 different phase history data availability levels: 100%, 87.89%, 76.56%, 56.25%, 25%
- 2 different noise levels ($\sigma_n \in \{0.1, 1\} \sigma_y$)
- Rectangular band-limitation for data reduction
Qualitative Results: Image 7, $\sigma_n = 0.1\sigma_y$ and full data
Qualitative Results: Image 7, $\sigma_n = 0.1\sigma_y$ and 87.89% data

Ground truth

FFT-based

FE-based [3]

PnP-based
Qualitative Results: Image 7, $\sigma_n = 0.1\sigma_y$ and 76.56% data
Qualitative Results: Image 7, $\sigma_n = 0.1\sigma_y$ and 56.25% data

Ground truth

FFT-based

FE-based [3]

PnP-based
Qualitative Results: Image 7, $\sigma_n = 0.1\sigma_y$ and 25% data

Ground truth

FFT-based

FE-based [3]

PnP-based
# Quantitative Results

Table: SNRs for selected images at various noise & data availability levels

<table>
<thead>
<tr>
<th>Available Data</th>
<th>Method</th>
<th>SNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$0.1\sigma_y$, Image 7</td>
</tr>
<tr>
<td>100%</td>
<td>FFT-based Reconstruction</td>
<td>36.539</td>
</tr>
<tr>
<td></td>
<td>Feature-enhanced Regularization [3]</td>
<td>36.968</td>
</tr>
<tr>
<td></td>
<td>Proposed Framework</td>
<td>38.075</td>
</tr>
<tr>
<td>87.89%</td>
<td>FFT-based Reconstruction</td>
<td>10.960</td>
</tr>
<tr>
<td></td>
<td>Proposed Framework</td>
<td>36.754</td>
</tr>
<tr>
<td>76.56%</td>
<td>FFT-based Reconstruction</td>
<td>8.659</td>
</tr>
<tr>
<td></td>
<td>Proposed Framework</td>
<td>35.465</td>
</tr>
<tr>
<td>56.25%</td>
<td>FFT-based Reconstruction</td>
<td>6.393</td>
</tr>
<tr>
<td></td>
<td>Feature-enhanced Regularization [3]</td>
<td>7.886</td>
</tr>
<tr>
<td></td>
<td>Proposed Framework</td>
<td>25.199</td>
</tr>
<tr>
<td>25%</td>
<td>FFT-based Reconstruction</td>
<td>3.951</td>
</tr>
<tr>
<td></td>
<td>Proposed Framework</td>
<td>13.579</td>
</tr>
</tbody>
</table>
Preliminary Results on Real Scenes from TerraSAR-X

- Training based on the Netherlands Rotterdam Harbor Staring Spotlight SAR image (1041 × 1830)
  - Split into 448 non-overlapping 64 × 64 “windows”
  - 1075648 overlapping 16 × 16 patches extracted from windows
  - Patches augmented with rotations of 90°, 180°, 270°

- Test set: 751 selected windows extracted from the Panama High Resolution Spotlight SAR image (2375 × 3375)
Qualitative Results: Image 608, $\sigma_n = \sigma_y$, full data

Reference image

FFT-based

FE-based [3]

PnP-based
Qualitative Results: Image 608, $\sigma_n = \sigma_y$, 87.89% data
Qualitative Results: Image 608, $\sigma_n = \sigma_y$, 76.56% data
Qualitative Results: Image 608, $\sigma_n = \sigma_y$, 56.25% data

Reference image

FFT-based

FE-based [3]

PnP-based
Qualitative Results: Image 608, $\sigma_n = \sigma_y$, 25% data

Reference image

Conclusion

• A line of inquiry that involves:
  – Radar sensing
  – Computational imaging
  – Signal representation, compressed sensing
  – Machine learning

• Sparsity is a useful asset for radar imaging especially in nonconventional data collection scenarios (e.g., when the data are sparse, irregular, limited)

• Deep learning methods may have the potential to learn complicated spatial patterns and enable their incorporation as priors into computational radar imaging

• Radar imaging offers a rich set of inference problems that can benefit from machine learning
  – Reflectivity field, anisotropy, model (parameters), object motion
  – Nonlinear case, multiple scattering, multi-static imaging, passive imaging

• Several challenging yet potentially rewarding themes for the near future:
  – Confidence/uncertainty characterization, posterior sampling
  – Machine learning for computational imaging and/or decision making